A GENERALISED SCATTERING MATRIX REPRESENTATION OF SLOT RADIATORS EXCITED BY A NRD GUIDE

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by
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CERTIFICATE

This to certify that the work presented in this thesis ttitled "A GENERALISED SCATTERING MATRIX REPRESENTATION OF SLOT RADIATORS EXCITED BY A NRD GUIDE" has been carried out by Miss C.Ramani under my supervision and the same has not been submitted elsewhere for a degree.

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CONTENTS

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Recent emphasis in Antenna Array research has been in the low cost design employing printed circuit technology for fabrication, especially at high end of the Microwave spectrum and MilliMeter(mm) wave frequencies. There have been studies of planar Slot Arrays fed by various types of transmission lines, such as, Rectangular Waveguide, Strip line, Microstrip etc.Ref[8, 7]. While waveguide fed Slot Arrays are machined, the Microstrip and Strip Line fed Slots are easily fabricated using printed circuit technology.

One of the major problems in such designs is the losses in the feed network, especially at mm wave frequencies. At mm wave frequencies the conventional Strip Line and Microstrip transmission lines have significant losses and other difficulties in fabrication. even for, integrated circuit applications, which has resulted in researchers looking for alternate types of transmission lines such as Fin Lines, Non-Radiative dielectric waveguide(NRD), Suspended Strip Lines, Groove Guide etc. The NRD guide, the transmission line considered in this thesis, consists of a rectangular low loss dielectric strip sandwiched between two conducting plates.fig[1.1]. The most useful mode of propagation in the NRD guide, the PM_{11} mode, has low loss property. In this thesis, the study of Slot radiators fed by NRD guide has been attempted with the design of a planar Slot array fed by NRD in mind.

A planar array has several aspects. First the design of array factor or the antenna pattern synthesis. This gives us the excitation coefficient of the individual Slots in an array that yield the desired radiation pattern. The second portion is to design a resonant Slot with the desired band width and feed network. This is a very involved problem because, in an array environment the Slot behaves differently than in an isolated single Slot environment, because of the mutual coupling among the Slots and the interaction with the feed network, matching structures etc. Therefore it is necessary to characterize each Slot fed by the NRD in the array environment. It is necessary to evaluate the power coupled to each Slot, coupling between the Slots and the internal coupling through the transmission line, among the Slots when the feeding structure is not isolated, as in a linear array of Slots on the ground plane of a NRD.

The present problem of Slots fed by NRD is further complicated by the fact that the NRD can propagate more than one mode and we have to account for the interaction between these modes due to the presence of the Slot discontinuity in the transmission line. Each of these modes also propagate with different phase constant which also has to be taken into account in designing the excitation of the Slots in the array.

1.2 PROBLEM DEFINITION

The entire design of an array is beyond the scope of this thesis. What is attempted here is the characterisation of a mutually coupled pair of Slots fed by NRD. The three major propagating modes of the NRD are considered and an equivalent scatter matrix representation for the coupled pair of Slots is developed. The evaluation of the

scatter matrix elements, which are functions of the position and orientation of the Slots with respect to the guide center line, involves integration over the guide and Slot cross section

This thesis discusses the analysis of NRD guide for its modes and design curves are provided to select the NRD dimensions for given frequency band of operation. The field structure of the NRD modes is necessary for the analysis of Slot cut in the conducting wall of the guide.

A scatter matrix equivalent of the individual Slot is developed first. Although there are physically only two ports, the multi-modes propagating in the NRD, and interaction between them due to the presence of the Slot discontinuity is taken into account by developing a six port scatter matrix equivalent. When two Slots are present on the same guide the equivalent circuit would be two such matrices interconnected by transmission line sections of different lengths for ports corresponding to each mode. This accounts for the internal coupling between the Slots. The external mutual coupling is also converted to three isolated ports and the coupling is taken into account in an external mutual coupling scattering matrix of 6-port. The external mutual coupling scatter matrix will have 3n ports for n-slot array, whereas the individual slot nine port scatter matrix remains the same for each slot.

Once this model is obtained, matrix manipulation methods are available to find the overall scatter matrix of an n-element linear array. The parameters of interest are power radiated by each element in an array and the input reflection coefficient. All this is analysis, i.e given an NRD and Slot geometry, we can calculate the equivalent s-matrices and find the input reflection coefficient and

the radiation pattern. The INVERSE problem of synthesis is not an easy one.i.e given the array excitation coefficients, synthesize the scatter matrix elements and from them obtain the Slot position, orientation and dimensions. However analysis can be used in a trial and error method of design.

1.3 NRD AND NRD FED SLOT.

If dielectric strips of proper dielectric constant are introduced between parallel plates separated by a distance comparable to half a wavelength, the waves are able to propagate freely along the strips, whether they be straight or curved. Radiated waves if any, decay outside the strips. This is the principle of operation of the NRD guide. this guide differs from the H-guide[7], in the sense that, in H-guide the side wall separation is made much larger than half a wavelength to realize low loss propagation, rather than to suppress radiated waves.

The NRD guide has many valuable properties. The attenuation of the dominant PM_{11} mode is much lower compared to the rectangular waveguide and it decreases with increase in frequency. A wave trapped in the dielectric strip propagates in the z-direction. The total resulting field extends to the air-filled region, but is confined to the space between the parallel plates. The field decays exponentially in these air- filled regions, but propagates in z-direction only. The promise of NRD guide lies in this non-radiating property. Because of this, bends and discontinuities can be easily incorporated in complicated integrated circuits. Though, Microstrip, slot lines and Fin lines are in common usage at lower end of mm wave frequencies, NRD guide is superior because of its low loss characteristics. In the

other structures, conduction losses increase as frequency of operation increases.

NRD guide supports two kinds of hybrid modes PM_{mn} and PE_{mn} in addition to the TE_{m0} modes. PM modes stand for magnetic fields parallel to air-dielectric interface and PE modes stand for electric fields parallel to air-dielectric interface. Further, the modes are divided into even and odd modes.The TE_{m0} modes are degenerate PE_{mn} modes. For these modes there is no variation of fields between the conducting plates. Because of the constant field distribution in the y-direction, for n=0 order modes, the transverse magnetic fields i.e TM modes cannot exist in a NRD guide transmission structure.

For the successful operation of the NRD guide, it has to be designed, so that only PM_{11} mode propagates. But the NRD dimensions required to propagate PM_{11} mode, also supports PE_{11} and TE_{m0} modes. Except TE_{10} mode, the other higher order TE_{m0} modes can be ignored because of the guide symmetry. However the NRD guide can be excited to propagate PM_{11} mode only. The NRD guide can be excited using other type of transmission lines. For this, a transition to NRD guide is required. A transition from rectangular to NRD guide is fairly simple. A coaxial line to NRD or strip line to NRD are very useful in many applications. These transitions can be designed to excite only PM_{11} mode, although the guide can support other modes.

Cutting Slots in the metal wall of the NRD guide involves in unifying the feeding and radiating structures. This ensures a non-radiating transmission line, permits easy machining or etching of the Slots and provides a mechanically rigid structure. Usually Slots are arranged in arrays.In this thesis a linear array , with Slots having alternate inclinations is analysed. The Slots of general

interest are resonant Slots having a length of approximately $\lambda/2$ and a width small compared with the length of the Slot. A good approximation to the electric field distribution in the Slot whose length 2l is within 5% of resonance is

$$E = V_m/W \cos(\pi\zeta/2l)\ \hat{\eta} \qquad 1.1$$

where, 2l is the length of the Slot,

W is the width of the Slot

V_m is the voltage at the center of the Slot

ζ , η are the Slot axis.

In other words the electric field distribution in the Slot can be assumed to be sinusoidal along its length and independent of the feeding system; and the direction of the E field is transverse to the length of the Slot[6,14].

The Slot discontinuity in the NRD guide causes the back and forward scattering of the incident mode power apart from radiating into free space if it intersects the current lines of the propagating mode.Ref[8] Considering that the NRD guide propagates only PM_{11} mode, power balance can be written which connects the Slots excitation to its orientation. But, an NRD cannot be designed for a single mode propagation.The three propagating modes PM_{11} , PE_{11} and TE_{10} modes are considered and a 6x6 scatter matrix is developed for the Slot discontinuity fed by an NRD guide.

The knowledge of the back and forward scatter mode amplitudes is sufficient to determine the scatter matrix elements. Further, assuming the network to be loss less and passive, the number of unknown elements to be determined reduces to a minimum. The relation between the backward and forward scattered coefficients gives the 2-port transmission line equivalent of the Slot in the NRD guide. To

account for radiation three more mutually isolated ports are added to the 6 port matrix. Using the unitary property of the scatter matrix the additional elements of the scatter matrix resulting due to the radiation ports are determined.

A Slot along the direction of propagation, cut longitudinally along the guide center line in the upper conducting plate of the guide, does minor perturbations to the current due to the PE_{11} mode. As a result very less power is coupled through the Slot. Similarly a Slot normal to the guide center line has negligible effect on the current lines due to PM_{11} mode. A linear Slot array with a spacing λ between the elements longitudinally placed along the guide center line gives rise to broadside array causing unwanted grating lobes. However, by using Slant Slots with alternate inclinations, coupling into the guide can be adjusted, while spacing the Slots at half a guide wavelength.

Mutual coupling between the Slots results due to both, internal and external conditions.A 6-port scatter matrix is developed to account for external mutual coupling between two Slots which is extended to an N-element linear Slot array. The internal coupling between the propagating modes is taken into account by cascading the individual 9-port scatter matrix of the Slot with transmission lines pertaining to the respective modes.

1.4 LITERATURE SURVEY

The NRD guide has been introduced by Yoneyama and Nishida[3]. It promises a very useful medium for realising mm wave integrated circuits. Ref[4,5] give the detailed uses of NRD guide and its properties, along with its prominence.

The NRD guide supports different kinds of propagating modes.

The field equations of these different modes, their characteristics and properties have been reported in a book by Bhartia and Bahl[8]. But the data provided in this book is insufficient for NRD design. Sufficient information regarding NRD guide analysis, design curves and its excitation has been done by D.Dawn[19]. But his thesis deals with a single approach which is quite lengthy. This thesis on the other hand gives the Transverse Resonance Technic(TRT) method, with which determination of the characteristic equations and dispersion characteristics is fairly simple.

Marvin Cohn[17] and R.A.Moor & R.E.Beam[18] have also discussed the method of analysis for the modes in the NRD guide, but each have their own setbacks. In the first case, M.Cohn confined himself to TE modes, whereas the approach used by R.A.Moor & R.E.Beam is very laborious.

The design of Slot array in the NRD guide has been published by J.A.G.Malherbe[1]. In this paper, the author has taken into consideration the dominant PM_{11} mode only. But, PE_{11} and TE_{10} modes cannot be ignored, because of their dispersion characteristics and therefore their contribution cannot be termed as negligible. This thesis tries to overcome the limitation by building a generalised scatter matrix taking into account the three propagating modes PM_{11}, PE_{11} and TE_{10} modes.

The generalised scattering matrix technique given in the book by Itoh[12] has been used to develop, the scattering matrix of the slot in the NRD guide. The approach suggested by R.S.Elliot[8] has been used to calculate the matrix elements of the Generalised Scatter Matrix. The book by Amitay, Galindo & Wu [11] was also very helpful in giving insight into the problem.

1.5 ORGANISATION OF THE THESIS

The analysis of the NRD guide fields using the TRT approach and the conventional method is discussed in chap 2. the field expressions for the various NRD modes, their properties and characteristic equations are also given in chap2. A complete design example of NRD guide is described and the attenuation characteristics of PM_{11}, PE_{11} and TE_{10} modes ia also provided.

chap 3 has the analysis of a single Slot and a Scatter Matrix is developed or an isolated Slot fed by the NRD guide. The elements of the scatter matrix are determined using the back and forward scattered mode amplitudes. An example at an operating frequency 95Ghz is given for Slot inclined at an angle of 60°.

Chap 4 takes into account the mutual coupling between the two Slots. A scatter matrix is developed for the external mutual coupling which is the order of 6. the coupling between two Slots is extended to an N-element linear array. Both internal and external mutual coupling between the propagating modes has been discussed.

The summary of the work presented in this thesis and the scope for future work on the topic are the contents of Chap 5.

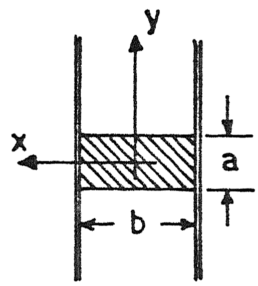


FIG.[1·1] NRD-waveguide cross section.

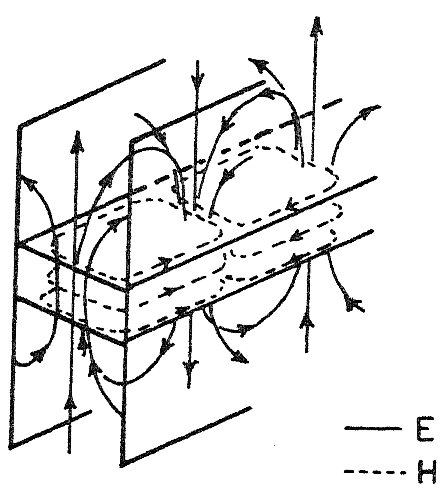


FIG.[1·2] Electric and magnetic field in NRD waveguide for PM_{11} mode.

CHAPTER 2

NONRADIATIVE DIELECTRIC GUIDE FIELDS AND CHARACTERISTICS

2.1 INTRODUCTION

The fields in the NRD guide can be obtained from the electric and magnetic hertizian potentials depending upon the mode under investigation. The NRD guide supports two kinds of surface wave modes PM_{mn} and PE_{mn}. These modes are hybrid in nature and can be represented as a combination of TE_{mn} and TM_{mn} modes, where m and n denote the half sinusoidal variations of the fields in the dielectric in the X and Y directions, respectively.

The designation PM stands for magnetic fields parallel to the air-dielectric interface (i.e.H_x = 0, refer to Fig[2.1] for co-ordinate system.), whereas, PE stands for electric fields parallel to the air-dielectric interface (i.e E_x = 0). Further, each class of mode can be either even symmetric (H_y for PM and E_y for PE modes are even functions of x), or odd symmetric (H_y for PM and E_y for PE modes are odd functions of x).

In addition to the hybrid modes PM_{mn} and PE_{mn} , the guide also supports TE_{m0} modes which do not have any variation of the fields in the Y-direction (i.e between the ground planes). But, the electric field in these modes is parallel to the air-dielectric interface , and in fact they are degenerate PE modes. For any two parallel conducting metal plates separated by distance which is comparable to half wavelength, there is no variation in the fields between the conducting plates for n=0 order modes, hence an NRD guide does not support TM

modes.

2.2 ANALYSIS OF NRD GUIDE CHARACTERISTICS

To determine the fields in the NRD guide two methods are applied:

i) The first approach is used in ref[7] and the compete analysis is given. In this approach, TE_{mn} and TM_{mn} fields are determined and the air-dielectric boundary conditions of the guide are satisfied by the combination of these modes.

For TE_{mn} fields $E_z=0$. The fields are derived from the magnetic hertizian potential $\Pi_h = \hat{z}\,\Pi_{hz}$, using the equations

$$E = -j\omega\,\mu\,\nabla \times \Pi_h$$

$$H = \nabla\times \nabla\times \Pi_h$$

Where Π_h is the solution of the homogeneous Helmholtz equation,

$$\nabla^2\Pi_h + k^2\Pi_h = 0$$

$$\Pi_h = \hat{z}\,\psi_h(x,y)\,\exp(-\gamma\,z)$$

$$k = \omega^2\mu\varepsilon$$

Similarly for TM_{mn} fields $H_z=0$ and the fields are obtained from an electric hertizian potential $\Pi_e = z\,\Pi_{ez}$ using the the equations

$$E = \nabla \times \nabla \times \Pi_e$$

$$H = j\omega\,\varepsilon\,\nabla \times \Pi_e$$

Where $\Pi_e = \hat{z}\,\psi_e(x,y)\,\exp(-\gamma\,z)$

Πe is also a solution of the scalar helmholtz equation.

Except for TE_{m0} modes, The remaining TE_{mn} and TM_{mn} modes do not satisfy the air-dielectric interface boundary conditions individually. But, nevertheless a combination of TE_{mn} and TM_{mn} modes can be selected to satisfy the boundary conditions which represent the hybrid PE_{mn} PM_{mn} modes.

ii) In the second approach, the NRD fields are determined

using the TRANSVERSE RESONANCE TECHNIQUE (TRT). Since PE_{mn} and PM_{mn} modes have $E_x=0$ and $H_x=0$ respectively, they can be looked at as TE to x and TM to x directions and the field components can be expressed in terms of a x-directed Hertz vector potential. Then, PEmn mode solution can be obtained by assuming a magnetic type Hertizian potential of the form

$$\Pi_h = \psi_h(x,y) \exp(-\gamma z) \hat{x}$$

and PM_{mn} mode solution can be obtained in terms of the electric type Hertizian potential of the form

$$\Pi_e = \psi_e(x,y) \exp(-\gamma z) \hat{x}$$

Appropriate form for the functions ψ_h and ψ_e in the separable form are assumed in the air and dielectric regions. The propagation constants can be obtained by a transmission line equivalent in x-direction and imposing the resonance condition in the transverse equivalent circuit shown in the Fig[2.3].

Since the NRD is assumed to be made of infinite conducting plates, in the equivalent circuit the input impedance is same as the wave impedance for the transmission line sections on the left and right of the dielectric loaded portion. therefore, jX_1 is the transverse wave impedance of the TE to x or TM to x mode under consideration. In order for such a mode to propagate along Z-direction with no attenuation it must be decaying outside the dielectric region to zero.

In sec(2.1) the hybrid modes PM_{mn} and PE_{mn} are classified as even symmetric and odd symmetric depending on the field components Hy and E_y respectively. Applying the same condition to the Fig[2.3] in TRT, the characteristic equations for these modes are derived.

In the Fig[2.4], the plane of symmetry is X=0 and the condition

of resonance is applied at this plane. For TM to x modes i.e PM_{mn} modes, the transverse equivalent circuit is obtained by taking the wave impedance as the characteristic impedance Z_{o1} and Z_{o2}. X_1 is Z_{o1}. Hence,

$$X_1 = k_{x2}/\omega\,\varepsilon_0 \quad \text{and} \quad Z_{o1} = k_{x1}/\omega\,\varepsilon_0\varepsilon_r$$

Moreover for TM to x modes, the plane of symmetry X=0 is an electric wall for even symmetry (i.e even modes) and a magnetic wall for odd symmetry (i.e odd modes). The input impedance looking right, at the X=0 plane is

$$Zin = Zo(Z_l + jZo\tan(k_{x1}a))/(Zo + jZ_l\tan(k_{x1}a)) \qquad 2.1$$

where $Zo = Z_{o1}$ and $Z_l = -jX_1$

The resonance condition for even modes is Zin = 0.

Therefore, $Z_l + jZo\tan(k_{x1}a) = 0$

$$-k_{x1}/\omega\,\varepsilon_0 + k_{x2}/\omega\,\varepsilon_0\varepsilon_r\,\tan(k_{x1}a) = 0$$

$$k_{x1}\tan(k_{x1}a) = \varepsilon_r k_{x2} \qquad 2.2$$

Resonance condition for odd modes is, Zin = ∞

Therefore, $Zo + jZ_l\tan(k_{x1}a) = 0$

$$k_{x1}\cot(k_{x1}a) = -\varepsilon_r k_{x2} \qquad 2.3$$

For TE to x modes(i.e PE_{mn} modes), the symmetry plane X=0 in an electric wall for odd symmetry (i.e odd modes) and a magnetic wall for even symmetry (i.e even modes)

From equn. (2.1), for odd modes Zin = 0.

$$Z_l + jZo\tan(k_{x1}a) = 0$$

But, in this case $Z_l = j\omega\,\mu_0/k_{x2}$ and $Zo = \omega\,\mu_0/k_{x1}$

Therefore, $k_{x1} + k_{x2}\tan(k_{x1}a) = 0$

$$-k_{x2} = k_{x1}\cot(k_{x1}a) \qquad 2.4$$

For even modes, Zin = ∞

$$Zo + jZ_l\tan(k_{x1}a) = 0$$

$$k_{x2} = k_{x1}\tan(k_{x1}a) \qquad 2.5$$

The characteristic equations (2.2) to(2.5) obtained using the TRT method are same as those obtained using the conventional method.

2.3 NRD GUIDE FIELDS

The different mode fields that exist in a NRD guide are given in the tables I,II,III. The various parameters involved in the field expressions and the characteristic equations for the modes are obtained by applying the boundary conditions at the air-dielectric interface.

The characteristic equations relate, the transverse phase constant in the dielectric filled region k_{x1}, and the transverse attenuation constant in the free space region k_{x2}. k_{x2}, must be positive for all the modes because in the NRD guide structure, the fields in region(2) and region(3) of Fig[2·1] must decay to zero.

i) Characteristic Equations for the NRD modes.

PM^{e}_{mn}	(m odd)	$\varepsilon_r k_{x2} = k_{x1}\tan(k_{x1}a)$	2.6
PM^{o}_{mn}	(m even)	$-\varepsilon_r k_{x1} = k_{x1}\cot(k_{x1}a)$	2.7
PE^{e}_{mn}	(m odd)	$k_{x2} = k_{x1}\tan(k_{x1}a)$	2.8
PE^{o}_{mn}	(m even)	$-k_{x2} = k_{x1}\cot(k_{x1}a)$	2.9
TE^{e}_{mo}	(m odd)	$k_{x2} = k_{x1}\tan(k_{x1}a)$	2.10
TE^{o}_{mo}	(m even)	$-k_{x2} = k_{x1}\cot(k_{x1}a)$	2.11

Using the condition of separability, the transverse wave propagation constant can be written in terms of longitudinal propagation constant and it is given by the expression

$$k_{x1}^2 = k_o^2\varepsilon_r - \beta^2 - (n\pi/b^2) \qquad 2.12$$

$$k_{x2}^2 = -k_o^2 + \beta^2 + (n\pi/b)^2 \qquad 2.13$$

$$k_o^2 = \omega^2\mu_0\varepsilon_0$$

$$k_i^2 = \omega^2\varepsilon_0\varepsilon_i - \beta^2 \qquad \text{for } i=1,2,3$$

TABLE I: NRD FIELD EQUATIONS FOR PM_{mn} MODES

FIELD COMPONENT	EVEN MODES $Hy(x)=Hy(-x)$	ODD MODES $Hy(x)=-Hy(-x)$
E_{x1}	$jC(ZW_1+(\beta^2k_{x1}^2/Z)W_2)$ CX SY	$jCQ(ZW_1+(\beta^2k_{x1}^2/Z)W_2)$ CX SY
E_{x2}	$jCQ(ZW_1Ca-\beta^2(k_{x1}k_{x2}/Z)W_2Sa)$ E_1 SY	$jDQ(ZW_1Sa+\beta^2(k_{x1}kx2/ZW_2Ca)$ E_1 SY
E_{x3}	$jCQ(ZW\ Ca-\beta^2(k_{x1}k_{x2}/Z)W\ Sa)$ E_2 SY	$-jDQ(ZW_1Sa+\beta^2k_{x1}k_{x2}/Z)W_2Ca)$ E_2 SY
E_{y1}	$-jCk_{x1}k_1^2W_2$ SX CY	$jDk_{x1}k_1^2W_2$ CX CY
E_{y2}	$-jCQ(W_1k_{x2}Ca-\beta^2k_{x1}W_2Sa)$ E_1 SY	$-jDQ(W_1k_{x2}Sa+\beta^2k_{x1}W_2Ca)$ E_1 SY
E_{y3}	$jCQ(W_1k_{x2}Ca-\beta^2k_{x1}W_2Sa)$ E_2 SY	$-jDQ(W_1k_{x2}Sa+\beta^2k_{x1}W_2Ca)$ E_2 SY
E_{z1}	Bk_1^2 SX SY	Ak_1^2 CX SY
E_{z2}	$Bk_1^2\ E_1Sa$ SY	$Ak_1^2\ E_1Ca$ SY
E_{z3}	$-Bk_1^2\ E_2Sa$ SY	$Ak_1^2\ E_2Ca$ SY
H_{y1}	$jC\beta k_1^2/Z$ CX SY	$jD\beta k_1^2/Z$ SX SY
H_{y2}	$jC\beta Q(ZCa\ -(k_{x1}k_{x2})/(\varepsilon_rZ)Sa)$ E_1 SY	$jD\beta Q(ZSa+(k_{x1}k_{x2})/(\varepsilon_rZ)Ca)$ E_1 SY
H_{y3}	$jC\beta Q(ZCa-(k_{x1}k_{x2})(\varepsilon_rZ)Sa)$ E_2 SY	$-jD\beta Q(ZSa+(k_{x1}k_{x2})/(\varepsilon_rZ)Ca)$ E_2 SY
H_{z1}	Ck_1^2 CX CY	Dk_1^2 SX CY
H_{z2}	$Ck_1^2E_1$ Ca CY	$Dk_1^2E_1$ Sa CY
H_{z3}	$Ck_1^2E_2$ Ca CY	$-Dk_1^2E_2$ Sa CY

$B = -C(\beta k_{x1}W_2)/Z$ $\qquad$ $A = D\ (\beta k_{x1}W_2)/Z$

TABLE II: NRD FIELD EQUATIONS FOR PE_{mn} MODES

FIELD COMPONENT	EVEN MODES $E_y(x)=E_y(-x)$	ODD MODES $E_y(x)=-E_y(-x)$
E_{y1}	$jDW_1\ k_1^2/k_{x1}$ CX CY	$-jCW_1\ k_1^2/k_{x1}$ SX CY
E_{y2}	$jDk_1^2\ W_1/k_{x1}$ Ca E_1 CY	$-jCk_1^2\ W_1/k_{x1}$ Sa E_1 CY
E_{y3}	$jDk_1^2\ W_1/k_{x1}$ Ca E_2 CY	$jCk_1^2\ W_1/k_{x1}$ Sa E_2 CY
E_{z1}	Ak_1^2 CX SY	Bk_1^2 SX SY
E_{z2}	Ak_1^2 Ca E_1 SY	Bk_1^2 Sa E_1 SY
E_{z3}	Ak_1^2 Ca E_2 SY	$-Bk_1^2$ Sa E_2 SY
H_{x1}	$jA(Z/W_2+\ \beta^2k_{x1}^2/W_1Z)$ Cx CY	$jC(\beta k_{x1}+\ k_o^2\varepsilon_r Z^2/(\beta k_{x1}))$SX CY
H_{x2}	$jAQ(\omega\ \varepsilon_o Z-(\beta^2k_{x2}^2)/ZW_1)CaE_1CY$	$jCQ/(\beta k_{x2})(k_0^2Z^2-\beta^2k_{x2}^2)SaE_1CY$
H_{x3}	$jAQ(\omega\ \varepsilon_o Z-(\beta^2k_{x2}^2)/ZW_1)CaE_2CY$	$-jCQ/(\beta k_{x2})(k_0^2Z^2-\beta^2k_{x2}^2)SaE_2CY$
H_{y1}	$jAk_{x1}\ k_1^2/W_1$ SX SY	$-jC\ k_1^2/(\beta Z)$ XC SY
H_{y2}	$jAk_{x2}\ k_1^2/W_1$ Ca E_1 SY	$jCk_{x2}\ (k_1^2Z)/(\beta k_{x1})$ Sa E_1 SY
H_{y3}	$-jAk_{x2}\ k_1^2/W_1$ Ca E_2 SY	$jCk_{x2}\ (k_1^2Z)/(\beta k_{x1})$ Sa E_2 SY
H_{z1}	Dk_1^2 SX CY	Ck_1^2 CX CY
H_{z2}	Dk_1^2 Sa E_1 Cy	Ck_1^2 Ca E_1 CY
H_{z3}	$-Dk_1^2$ Sa E_2 CY	Ck_1^2 Ca E_2 CY

$A = -D\ W_1Z/\beta k_{x1}$ $\qquad$ $B = C\ W_1Z/\beta k_{x1}$

TABLE III: NRD FIELD EQUATIONS FOR TE_{mn} MODES

FIELD COMPONENT	EVEN MODES $E_y(x)=E_y(-x)$	ODD MODES $E_y(x)=-E_y(-x)$
E_{y1}	$jDW_1\ k_{x1}CX$	$-jCW_1\ k_{x1}\ SX$
E_{y2}	$-jDW_1\ k_{x2}Q\ Sa\ E_1$	$-jCW_1\ k_{x2}Q\ Ca\ E_1$
E_{y3}	$-jDW_1\ k_{x2}Q\ Sa\ E_2$	$jCW_1\ k_{x2}Q\ Ca\ E_2$
H_{x1}	$-jD\ \beta k_{x1}\ CX$	$jC\ \beta k_{x1}\ SX$
H_{x2}	$jD\ \beta k_{x2}Q\ Sa\ E_1$	$jC\ \beta k_{x2}Q\ Ca\ E_1$
H_{x3}	$jD\ \beta k_{x2}Q\ Sa\ E_2$	$-jC\ \beta k_{x2}Q\ Ca\ E_2$
H_{z1}	$D\ k_1^2\ SX$	$C\ k_1^2\ CX$
H_{z2}	$D\ k_1^2\ Sa\ E_1$	$C\ k_1^2\ Ca\ E_1$
H_{z3}	$-D\ k_1^2\ Sa\ E_2$	$C\ k_1^2\ Ca\ E_2$

THE VARIOUS PARAMETERS INVOLVED IN THE TABLES I ,II ,III ,WHICH GIVE THE FIELD EXPRESSIONS OF THE DIFFERENT MODES IN A NRD GUIDE ARE:

$SX = \mathrm{Sin}(k_{x1}x)$ $\quad CX = \mathrm{Cos}(k_{x1}x)$

$Ca = \mathrm{Cos}(k_{x1}a)$ $\quad Sa = \mathrm{Sin}(k_{x1}a)$

$SY = \mathrm{Sin}((n\pi/b)y)$ $\quad CY = \mathrm{Cos}((n\pi/b)y)$

$Z = n\pi/b$ $\quad W_1 = \omega\ \mu_o$

$W_2 = 1/(\omega\ \varepsilon_o\varepsilon_r)$ $\quad E_1 = \exp(k_{x2}(a-x))$

$E_2 = \exp(k_{x2}(a+x))$ $\quad Q = k_1^2/k_2^2$

note: for even modes the subscript m *is odd and for odd modes it is even. There is no condition on the subscript* n.

As already reasoned the value of k_{x2} must be positive.To meet this requirement on k_{x2}, in the equns.(2.6) to (2.11), the roots of the characteristic equations, lie in specified quadrants,

$$(m-1)\pi/2 \leq k_{x1}a \leq m\pi/2 \qquad 2.14$$

With the aid of the equations (2.12),(2,13) and (2.14), it can be easily proved that the necessary condition for the m^{th} order mode to propagate is

$$\boxed{2a/\lambda_o > (m-1)/(2\,\varepsilon_r-1)} \qquad 2.15$$

There is a requirement to be met on the spacing "b" between the parallel conducting plates, for the modes to propagate. But, for TE_{mo} modes, there exists no such requirements on "b" and these modes propagate for all values of"b". Hence for modes with $n \geq 1$, b must exceed some critical value b_c, for the propagation constant β to be real. Therefore setting $\beta=0$ and $b=b_c$ in equns.(2.12) & (2.13) cut-off height can be determined.

ii) Expression for b_c(the cut-off height) for different modes

$$k_{x2}^2 = (n\pi/b_c)^2 - k_o^2 \qquad 2.16$$

Substituting the characteristic equations in equn.(2.16), the cut-off distance "b_c" between the two ground planes for the different modes is:

PM^e_{mn} modes (m odd) :

$$b_c/\lambda_o = \frac{n(\ 1 + \tan^2(k_{x1}a)/\varepsilon_r^2)^{1/2}}{2(\ 1 + \tan^2(k_{x1}a)/\varepsilon_r)^{1/2}} \qquad 2.17$$

PM^o_{mn} modes (m even) :

$$b_c/\lambda_o = \frac{n(\ 1 + \cot^2(k_{x1}a)/\varepsilon_r^2)^{1/2}}{2(\ 1 + \cot^2(k_{x1}a)/\varepsilon_r)^{1/2}} \qquad 2.18$$

PE^e_{mn} modes (m odd) :

$$b_c/\lambda_o = n/2(\varepsilon_r \sin^2 k_{x1}a + \cos^2 k_{x1}a)^{1/2} \qquad 2.19$$

PE^{o}_{mn} modes (m even :

$$b_c/\lambda_o = n/2(\varepsilon_r \cos^2 k_{x1}a + \sin^2 k_{x1}a)^{1/2} \qquad 2.20$$

Therefore, for a particular mode to propagate, the distance between the parallel ground planes should be so chosen that they exceed b_c, which is determined using the above expressions corresponding to different modes of the NRD guide.

ii) Expressions for k_{x1}, k_{x2} and β :

First k_{x1} is determined. Once K_{x1} is known, k_{x2} and β can be determined. Using equations (2.12) and (2.13) and the characteristic eqns.(2.6) to (2.11), the equations leading to the calculation for various modes of the NRD guide are :

PM^{e}_{mn}, m odd :

$$k_{x1}^2\ (\varepsilon_r^2 + \tan^2 k_{x1}a) = k_o^2\, \varepsilon_r^2\, (\varepsilon_r - 1) \qquad 2.21$$

PM^{o}_{mn}, m even :

$$k_{x1}^2\ (\varepsilon_r^2 + \cot^2 k_{x1}a) = k_o^2\, \varepsilon_r^2\, (\varepsilon_r - 1) \qquad 2.22$$

PE^{e}_{mn}, m odd :

$$k_{x1}^2\ (1 + \tan^2 k_{x1}a) = k_o^2\, (\varepsilon_r - 1) \qquad 2.23$$

PE^{o}_{mn}, m even :

$$k_{x1}^2\ (1 + \cot^2 k_{x1}a) = k_o^2\, (\varepsilon_r - 1) \qquad 2.24$$

TE^{e}_{mo}, m odd :

$$k_{x1}^2\ (1 + \tan^2 k_{x1}a) = k_o^2\, (\varepsilon_r - 1) \qquad 2.25$$

TE^{o}_{mo}, m even :

$$k_{x1}^2\ (1 + \cot^2 k_{x1}a) = k_o^2\, (\varepsilon_r - 1) \qquad 2.26$$

iii) Guided wavelength λ_g :

$\beta = 2\pi/\lambda_g$, $\quad k_o = 2\pi/\lambda_o$

Therefore, $$\lambda_g = \lambda_o/(\varepsilon_r - (n\lambda_o/2b)^2 - (k_{x1}/k_o)^2)^{1/2} \qquad 2.27$$

For TE_{mo}, modes

$$\lambda_g = \lambda_o/(\varepsilon_r - (k_{x1}/k_o)^2)^{1/2} \qquad 2.28$$

iv) Cut-off Frequency f_c:

At cut-off frequency, $\beta=0$

$$f_c = (1/2\pi)\left[\frac{(k_{x1}^2 + (\pi/b)^2)}{\mu_o\varepsilon_o\varepsilon_r}\right]^{1/2} \qquad 2.29$$

Fig[2.6] shows the f-β curves for the first few modes.

2.4 DISPERSION CHARACTERISTICS OF NRD GUIDE

These characteristics determine the field containment in the NRD guide and whether the guide is single moded or multi-moded. Using these characteristics the NRD guide can be designed for dominant mode operation and good field containment.

Eqn(2.11), shows that for the propagation of first order modes, the requirement on the dielectric slab width is 2a > 0. Hence, PE_{11} and PM_{11} and TE_{10} modes exist for even a small width "2a". From Fig[2.5] which shows the variation of cu-off height "b_c" for different modes, an NRD guide designed for the dominant PM_{11} mode also propagates PE_{11} and TE_{10} modes. The TE_{10} mode does not have any bound on "b_c". Hence, the NRD cannot be designed to propagate the desired PM_{11} mode alone.

2.5 ATTENUATION CHARACTERISTICS OF NRD GUIDE

In all physical transmission systems there is always a loss of power due to the conductor and/or dielectric material of the transmission line during the transmission. Therefore, it is necessary to determine the manner in which power is attenuated as it propagates along the transmission line. The currents flowing on the walls of the

guide which have finite conductivity, gives rise to ohmic losses.

The flow of energy is mainly through the dielectric region and the guide walls act as guide lines to the power flow. The choice of a good dielectric is always essential to maintain low dielectric losses.

The sum conductor loss constant α_c, and dielectric loss constant α_d, is the total attenuation loss constant in the NRD guide.These loss constants are determined from the eqns.

$$\left.\begin{aligned} \alpha_c &= P_p/\ 2P_z \\ \alpha_d &= P_d/\ 2P_z \end{aligned}\right\} \text{ nepers/unit length} \qquad \begin{aligned} &2.30 \\ &2.31 \end{aligned}$$

Where, P_p and P_d are power loss factors/unit length in the parallel ground planes and the dielectric material respectively, and P_z is the longitudinal power flow.

i) Losses in the Guide Propagating, PM_{11} mode:

The fields of PM_{11} mode do not have the H_x component of the magnetic field. E_x, H_y, and H_z components of the fields are even functions of x.

Thereforethe longitudinal power flow

$$P_z = \int_0^b \int_{-\infty}^{\infty} 1/2 \ \mathrm{Re}\ (E \times H^*) \ .\ z\ dS$$

$$= 1/2 \int_0^b \int_{-\infty}^{\infty} Ex\ Hy^*\ dx\ dy$$

$$= b/4 \left[FL/k_{x2} + GM/k_{x1}(2k_{x1}a - \sin 2k_{x1}a) \right] \qquad 2.32$$

Where the symbols F,L,G,M are given by the expressions:

$$F = jC\, k_1^2/k_2^2 \left[\omega\, \mu_o \pi/b \cos k_{x1}a - \beta k_{x1}k_{x2}/\omega\, \varepsilon_o\varepsilon_r \pi/b \sin k_{x1}a\right]$$

$$L = jC\, k_1^2/k_2^2 \left[\pi/b \cos k_{x1}a - k_{x1}k_{x2}/\varepsilon_r \pi/b \sin k_{x1}a\right]$$

$$G = jC \left[\omega\, \mu_o \pi/b + \beta^2 k_{x1}^2/\omega\, \varepsilon_o\varepsilon_r \pi/b\right]$$

$$M = jC\, \beta\, k_1^2/(\pi/b)$$

The power dissipated in the top and bottom ground planes due to the finite conductivity is:

$$P_p = Rs/2 \int_S |H_t|^2 dS$$

$$= RsC^2k_1^4/2\ (Q^{ii}/k_{x1} + 2(\cos^2 k_{x1}a)/\ k_{x2} \qquad 2.33$$

Where $Q^{ii} = 2k_{x1}a + \sin 2k_{x1}a$

The power dissipated in the dielectric due to the losses in the ielectric is

$$P_d = 1/2\ \omega\, \varepsilon_o\varepsilon_r \tan\delta \int_0^b \int_{-a}^{a} |Ex1|^2 + |Ey1|^2 + |Ez1|^2 dx\, dy$$

$$= 1/4\ \omega\, \varepsilon_o\varepsilon_r \tan\delta\ b/2k_{x1}(G^2Q + (C^2k_{x1}^2k_1^4)/\omega\, \varepsilon_0\varepsilon_r\ Q^i + B^2k_1^4Q^i)$$

Where $Q^i = 2k_{x1}a - \sin 2k_{x1}a$

In the above expressions the constants B and C are related by the eqn.

$$B = C\beta k_{x1}/\omega\, \varepsilon_0\varepsilon_r \pi/b$$

Substituting eqns.(2.32) to (2.34) in eqn.(2.31) determines the attenuation loss in the NRD guide.

ii) Losses in the Guide propagating PE_{11} mode

PE_{11} mode is orthogonal to PM_{11} mode . It does not have Ex component of electric field. The field components Hx, Ey, Ez are even functions of x.

The longitudinal power flow, is

$$P_z = 1/2 \int_0^b \int_{-\infty}^{\infty} \mathrm{Re}(-Ey\ Hx^*)\ dx\ dy$$

$$= -b/4\ (HN/2k_{x1}Q^{ii} + KP/k_{x2}) \qquad 2.35$$

Where, the constants H,N,K,P are governed by the following eqns.:

$$H = jD\omega\ \mu_0 k_1^2/k_{x1}$$

$$N = jA\ (\omega\ \varepsilon_0\varepsilon_r\pi/b + \beta^2 k_{x1}^2/\omega\ \mu_0\pi/b)$$

$$K = jDk_1^2\ (\omega\ \mu_0/k_{x1})\ \cos k_{x1}a$$

$$P = jA\ k_1^2/k_2^2\ (\omega\ \varepsilon_0\pi/b - \beta^2 k_{x2}^2/\omega\ \mu_0\pi/b)\cos k_{x1}a$$

The losses in the parallel coonducting ground planes are:

$$P_p = Rs/2 \int_S |H_t|^2\ dS$$

$$= RsD^2k_1^4/2\ (Q^i/k_{x1} + 2\sin^2 k_{x1}a/k_{x2}) +$$

$$Rs/2(A^2U^2Q^{ii}/k_{x1} + 2U'^2\cos^2(k_{x1}a)/k_{x2}) \qquad 2.36$$

where thevariables U and U' are given by:

$$U = (\omega\ \varepsilon_o\varepsilon_r\pi/b + (\beta^2 k_{x1}^2)/(\omega\ \mu_o\pi/b)$$

$$U' = k_1^2/k_2^2\ (\omega\ \varepsilon_o\pi/b - (\beta^2 k_{x2}^2)/(\omega\ \mu_o\pi/b)$$

$$A^2 = D^2\ (\omega\ \mu_o\pi/b)^2/(\beta^2 k_{x1}^2)$$

The powerloss due to the dielectric region (1) in the guide is:

$$P_d = 1/2\ \omega\ \varepsilon_0\varepsilon_r\tan\delta \int_0^b\int_{-a}^{a} (\ |Ey1|^2+|Ez1|^2)dx\ dy$$

$$= b/4\ \omega\ \varepsilon_0\varepsilon_r\ \tan\delta\ Q^{ii}/2k_{x1}(H^2 + A^2k_1^4) \qquad 2.37$$

Therefore, the attenuation losses due to PM_{11} mode can be calculated by substituting eqns.(2.35) to (2.37) in eqn.(2.31).

iii) Losses in the Guide for TE_{10} mode

TE_{10} mode , has only transverse E field components. The longitudinal field component Ez is zero.

The powerflow in the direction of propagation is :

$$P_z = -1/2 \int_0^b \int_{-\infty}^{\infty} Re(Ey\ Hx^*)\ dx\ dy$$

$$= b/4\ (D^2 k_{x1}^2 \omega\ \mu_0 \beta / k_{x1}\ Q^{ii} + 2RT/k_{x2}) \qquad 2.38$$

Where the symbols R and T are given by :

$$R = -jD\ \omega\ \mu_0 k_{x2} k_1^2 / k_2^2\ \sin k_{x1} a$$

$$T = jD\ \beta k_{x2} k_1^2 / k_2^2\ \sin k_{x1} a$$

The losses in the parallel ground planes due to the finite conductivity is :

$$P_p = Rs/2 \int_S |H_t|^2\ dS$$

$$= RsD^2 k_1^4 /2\ (Q^i / k_{x1} + 2\sin^2 k_{x1} a / k_{x2}) + Rs/2 (D^2 \beta^2 k_{x1} Q^{ii} + k_{x2}\ k_1^2 / k_2^2\ 2\sin^2 k_{x1} a) \qquad 2.39$$

The due to the dielectric slab are:

$$P_d = 1/2\ \omega\ \varepsilon_0 \varepsilon_r \tan\delta \int_0^b \int_{-a}^{a} |Ey1|^2 dx\ dy$$

$$= -b\omega^3 \varepsilon_0 \varepsilon_r \tan\delta\ \mu_0^2 k_{x1} D^2 / 4 \quad Q^{ii} \qquad 2.40$$

Therefore, substituting eqns.(2.38) to (2.40) in eqn.(2.31) the attenuation loss due to the TE_{10} mode is obtained. this mode propagates in the NRD guide for all values of "a" & "b".

This finally completes the determination of the attenuation loss in the guide due to the three modes PM_{11}, PE_{11} and TE_{10}, which propagate for the NRD design of 1^{st} order mode.

2.6 CHOICE OF NRD GUIDE DIMENSIONS

The condition for the m^{th} order mode to propagate is given by eqn.(2.15)

For m=1 order modes to propagate, the condition to be satisfied is:

$$2a/\lambda_o > 0 \qquad 2.41$$

and for m=2 order modes to propagate, the condition to be satisfied is:

$$2a/\lambda_o > 0.4703604 \qquad 2.42$$

Therefore, for m=1 order modes to propagate, the bound on the dielectric slab width, from eqns(2.41) and (2.42) is:

$$0 < 2a/\lambda_o \leq 0.4703604 \qquad 2.43$$

If $2a/\lambda_o$ is chosen such that it is equivalent to the upper limit then, maximum operating bandwidth is obtained.

Therefore, for the design od NRD guide to support PM_{11},$PE_{11}$and TE_{10} modes

$$a = 0.2351802\ \lambda o \qquad 2.44$$

The cutoff distance "b_c" between the ground planes is calculated using expressions in sec(2.3) depending on the mode required to propagate.For a given bandwidth this dimension is calculated at the lower cut-off frequency.

k_{x1},k_{x2},β are determined at an optimum frequency within the bandwidth using the appropriate characteristic equations given in the sub-sec of sec(2.3).Primarily, k_{x1} is computed using the NEWTON-RAPHSON technic, and then β,k_{x2} are computed using the value of k_{x1}.

i) Design Example

An NRD guide is designed to support PM_{11} mode, and operate in the frequency range 90Ghz to 100Ghz. The lower cut-off frequency is 90Ghz and the operating frequency is chosen to be 95Ghz and ε_r=2.13. For, maximum operating bandwidth, in the eqn.(2.44) λo is determined at the lower cut-off frequency.

Hence, $a = 0.783933mm$

The dielectric width of the NRD guide is:

$$2a = 1.567866mm$$

The cut-off height between the two parallel ground planes is computed from the eqn(2.19) at the lower cut-off frequency 90Ghz.

At the operating frequency 95Ghz,

$$b_c = 1.3375\text{mm}$$

i)The transverse propagation constant in region(1)of Fig[1.1]

$$k_{x1} = 1464.417 \text{ rad/mt}$$

ii)The transverse propagation constant in region(2) of Fig[1.1]

$$k_{x2} = 1528.076 \text{ rad/mt}$$

iii)The propagation constant

$$\beta = 884.379 \text{ rad/mt}$$

iv)Wavelength at the operating frequency, 95Ghz is

$$\lambda_o = 3.155743619\text{mm}$$

v)The guided wavelength is

$$\lambda_g = 7.104625681\text{mm}.$$

2.7 CONCLUSIONS

For a given dimension `a' of the NRD guide, `b_c' the cutoff distance between the parallel ground planes required to propagate PE_{11} mode is less than that required to excite Pm_{11} mode. TE_{10} mode does not have any cutoff height, it propagates for all distances `b_c' between the ground planes,refer Fig[2.5]. Therefore, an NRD guide designed in section(2.6) for PM_{11} mode excitation also propagates PE_{11} and TE_{m0} modes.

Fig[2.4] gives the NRD dimensions for a given lower cutoff frequency and an operating frequency in the range 40-120Ghz. The design example shows that $\lambda_g/\lambda_o>1$. It is also evident from Fig[2.7], that $\lambda_g/\lambda_o>1$ for hybrid modes, PM_{11}, PE_{11}. We can conclude that the

NRD supports fast waves as far as hybrid modes are concerned, Since the PM_{11} mode is the principal mode in this structure.

PM_{11} mode warrants attention because of its conductor loss which decreases with increase in frequency. From the attenuation characteristics Fig[2.10], Fig[2.10] it is deduced that PM_{11} mode has low-loss constants in the given frequency range. Though PE_{11} attenuation characteristics decrease with frequency they are very high compared to PM_{11} mode characteristics. Moreover PM_{11} mode characteristics are insensitive to frequency variation.

Fig[2.9], shows that the dispersion characteristics of the NRD guide is linear in the frequency range 10-100Ghz for PM_{11} mode excitation, satisfying the ideal characteristics. Fig[2.6] & [2.8], show that for a small frequency band the characteristics are slightly curved near lower cutoff frequency, but approach the ideal characteristics as frequency increases.

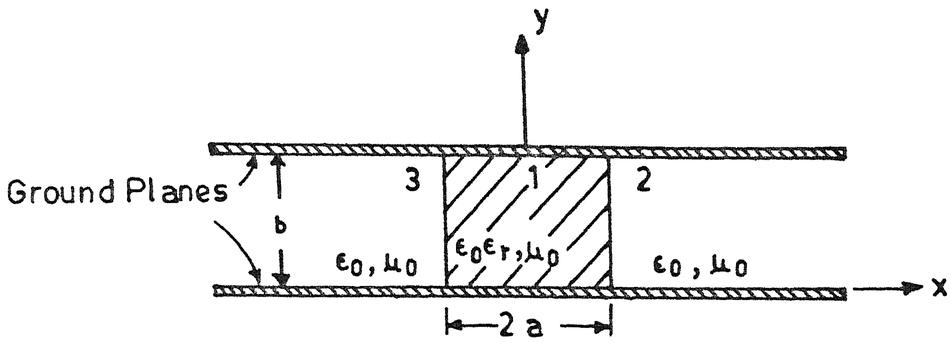


FIG. [2.1] NRD – guide geometry .

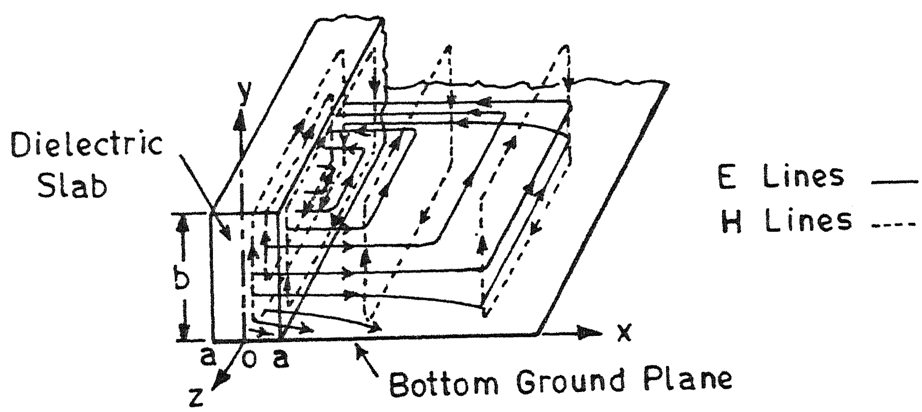


FIG. [2.2] Field configurations of the PM_{11} mode .

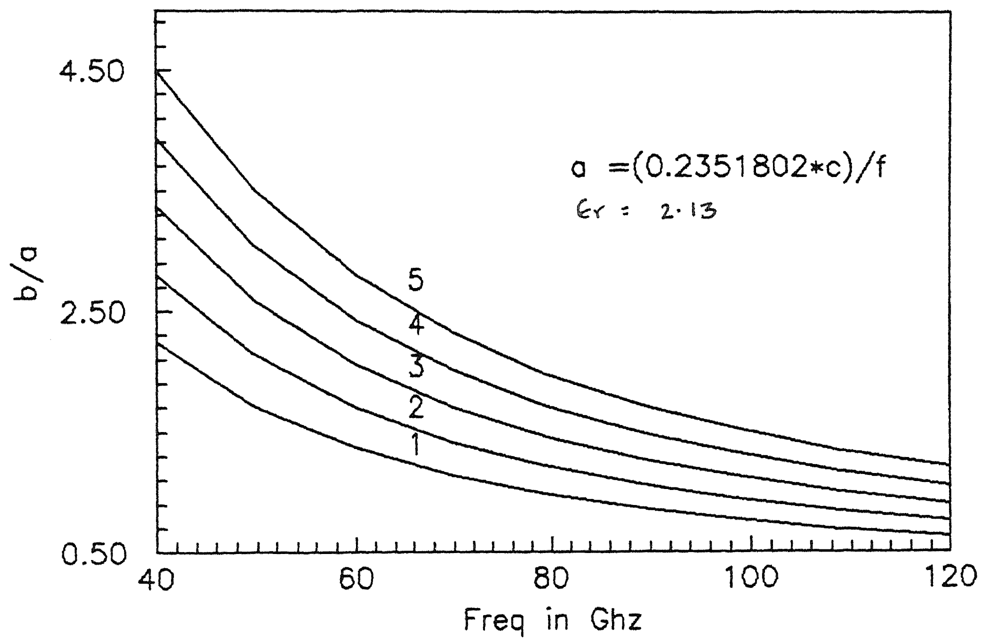
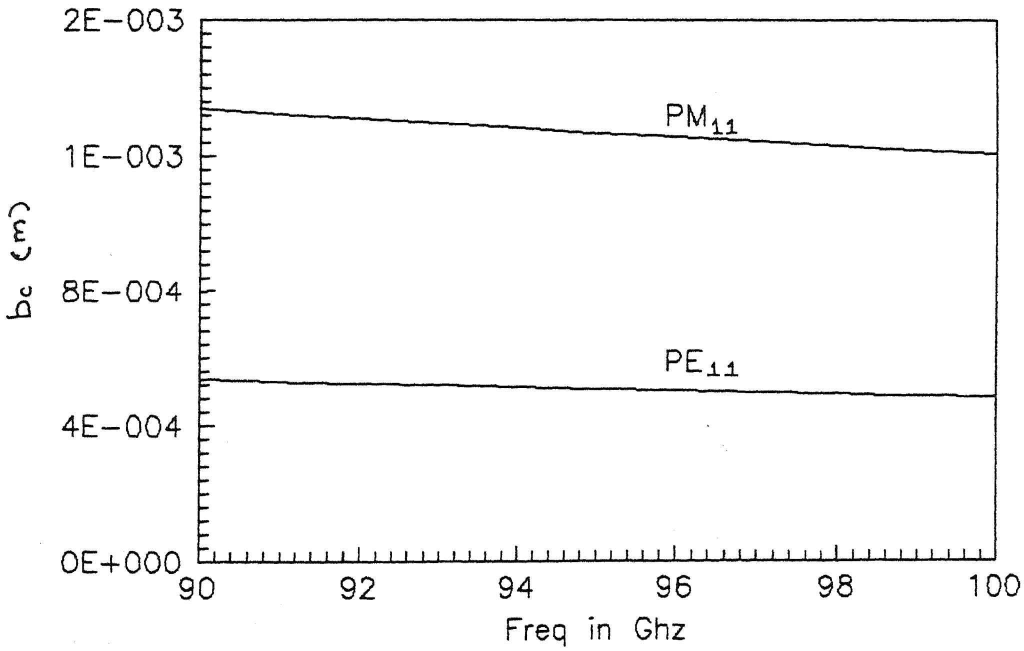


Fig [2.4]-Distance between the parallel plates Vs freq. (normalised)
Curves 1 to 5 : correspond to f = 90, 80, 70, 60, 50 GHz.



Fig[2.5] Distance between the parallel plates Vs freq.
(ϵ_r = 2.13, a = .0007839)

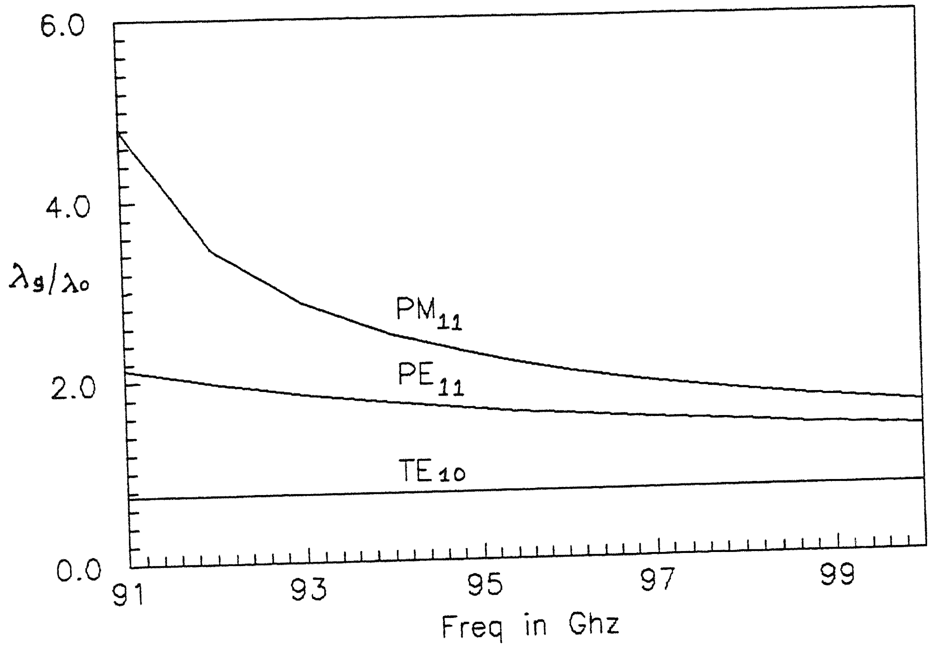


Fig [2·7] Variation of normalised guided wavelength with freq
(ϵ_r = 2·13 , b/a = 1·706)

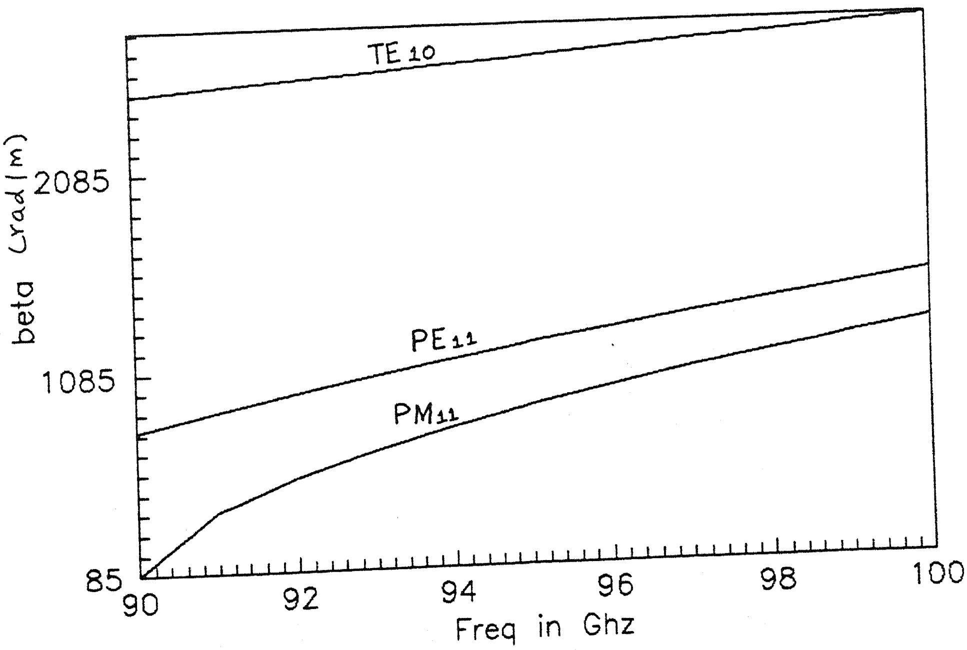


Fig [2·6] Dispersion Characteristics
(ϵ_r = 2·13 , b/a = 1·706)

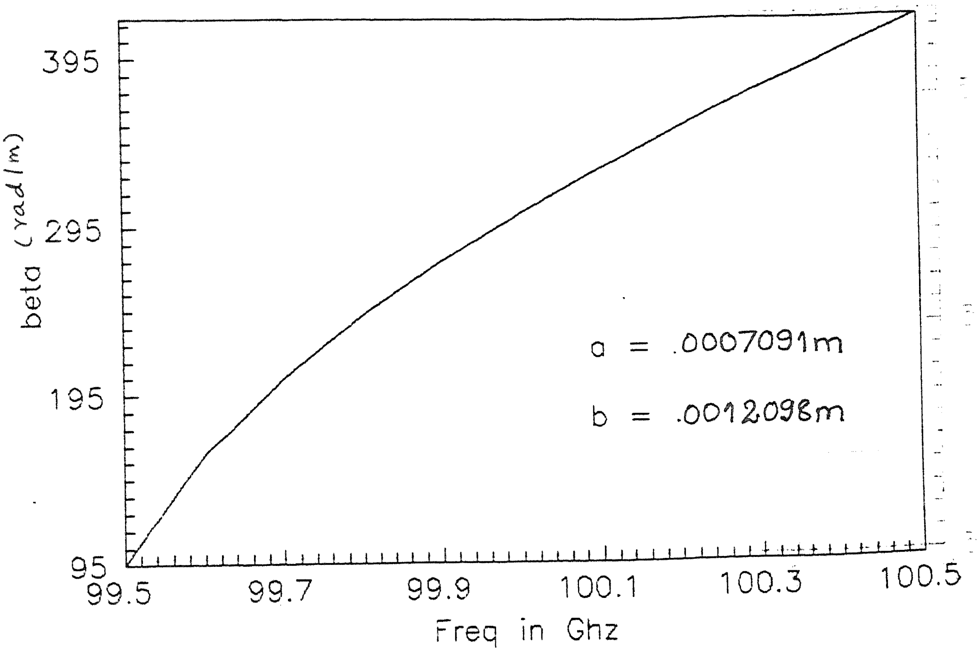


Fig [2.8] Dispersion Characteristics for PM_{11} mode
($\epsilon_r = 2.13$)

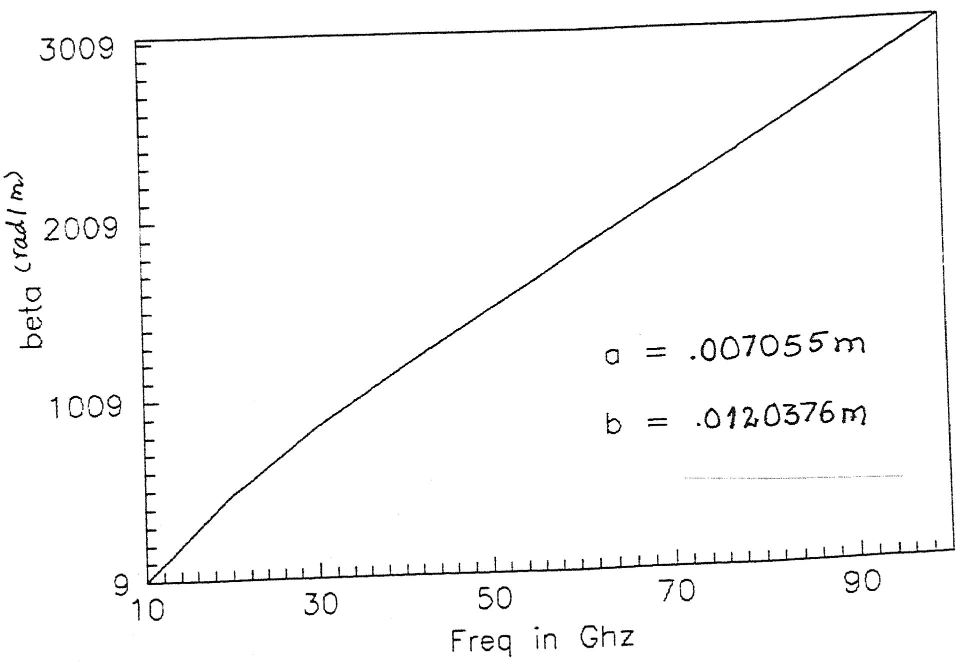
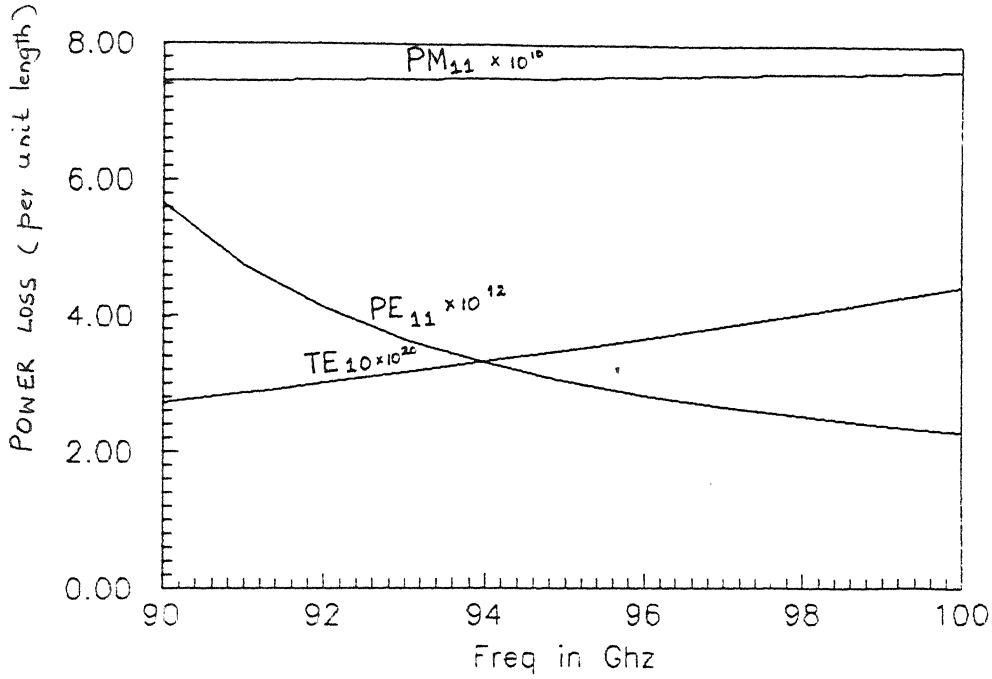
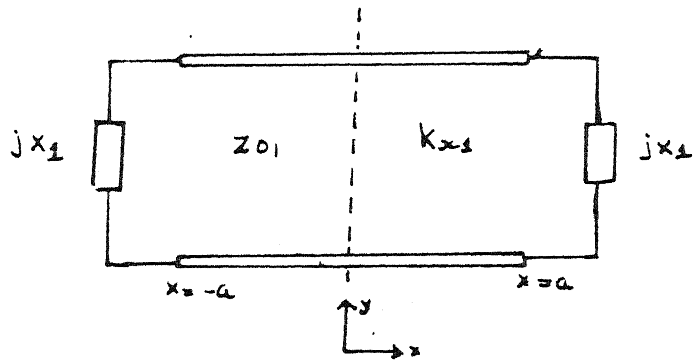


Fig [2.9] Dispersion Characteristics for PM_{11} mode
($\epsilon_r = 2.13$)



Fig[2.10] Attenuation Characteristics.
(ϵ_r = 2.13 , b/a = 1.706)



FIG[2.11] TRANSVERSE EQUIVALENT CIRCUIT OF NRD GUIDE

CHAPTER 3

ANALYSIS OF A SINGLE SLOT FED BY A NRD GUIDE

3.1 INTRODUCTION

A Slot in a guide carrying power in its modes will radiate , when it intersects the current lines or otherwise when it is excited.As already stated, mentioned in chapter 1, coupling can be adjusted by positioning of the slots or changing their orientation. The equivalent circuit that the Slot reduces to, in an equivalent transmission line representation is again a function of position of the Slot and its orientation. Under certain conditions, a Slot cut in a guide is equivalent to a series element, and in others it reduces to a shunt element. These circuit representations are obtained with the assumption that the guide supports a single mode.For multi-mode propagation, which is a more general case,the Slot fed by the guide is equivalent to a T or a Π-network .

3.2 THEORY OF A SLOT RADIATOR

. Let us consider a module with bounds Z_1, Z_2 and $Z_2>Z_1$, containing the Slot in the wall of an infinite guide as shown in Fig[3.1].If the guide is excited by a dominant mode i.e known field distribution, the slot scatters the field and as a result we have waves scattered in the forward direction i.e $Z>Z_2$ and backward scattered waves in the direction $Z<Z_1$ and some power is radiated. Hence the field in the guide can be written as

$$E_{1t} = \Sigma\ C_p\ E_{pt}\ \exp(-j\beta z) \qquad ,Z>Z_2 \qquad 3.1a$$

$$E_{1t} = \Sigma\ B_p\ E_{pt}\ \exp(-j\beta z) \qquad ,Z<Z_1 \qquad 3.1b$$

$$H_{1t} = \Sigma\ C_p\ H_{pt}\ \exp(-j\beta z) \qquad ,Z>Z_2 \qquad 3.1c$$

$$H_{1t} = -\Sigma\ B_p\ H_{pt}\ \exp(-j\beta z) \qquad ,Z<Z_1 \qquad 3.1d$$

The amplitude of the forward scattered waves Ca and the backward scattered waves Ba need not be equal, but they must be such that on super posing the two sets of waves afield is produced which matches the field over the slot and satisfies the general boundary conditions.

The next step is, to evaluate the scattered amplitude coefficients Cp and Bp. So, consider two fields $E_1 H_1$ and $E_2 H_2$ of the same frequency and both satisfying the homogeneous field equations. Let S, be a surface consisting of S_1 at $Z=Z_1$, S_2 at $Z=Z_2$ and S_3 is a surface which is skin tight against the walls of the guide (interior) between Z_1 and Z_2. With S, a source free region, application of reciprocity theorem connects the two fields by the relation

$$\int_S (E_1 \times H_2 - E_2 \times H_1)\ .\ dS = 0 \qquad 3.2$$

Let $(E_1 H_1)$ denote the scattered field due to the interaction of the slot and the incident mode. $(E_2 H_2)$ does not relate to the actual situation, nevertheless its use is essential to obtain the scattering coefficients. From equn(3.2) we have

$$\int_{slot} (E_1 \times H_2).dS = M_1 + M_2 \qquad 3.3$$

where

$$M_1 = \int_{S_1} (E_2 \times H_1 - E_1 \times H_2).\ dS_1 \qquad 3.4$$

$$M_2 = \int (E_2 \times H_1 - E_1 \times H_2) . dS_2 \qquad 3.5$$

The relation (3.3) is obtained , because on the sub-surface Ss tangential component of E_2 is zero and the tangential component of E_1 is zero everywhere on S_3 , except that part occupied by the slot.

The fields E_2 and H_2 in the guide can be written as

$$E_2 = E_{qt} \exp(-j\beta z) \qquad ,Z>Z_2 \qquad 3.6a$$

$$H_2 = H_{qt} \exp(-j\beta z) \qquad ,Z>Z_2 \qquad 3.6b$$

$$E_2 = E_{qt} \exp(-j\beta z) \qquad ,Z<Z_2 \qquad 3.6c$$

$$H_2 = H_{qt} \exp(-j\beta z) \qquad ,Z<Z_2 \qquad 3.6d$$

In equations (3.1)and (3.6), the subscripts p and q denote the half sinusoidal variations of the fields in the transverse directions,(i.e p=mn, q=m'n'), but because of the orthogonal properties of modes, the only contribution to M_1 and M_2 comes when p=q and in addition, the indices should refer to the same type of mode. Applying these conditions to equations (3.4)and (3.5), we get

$$M_1 = 2B_q \int (E_{qt} \times H_{qt}).dS_1 \qquad 3.7$$

and M_2 is equal to zero. 3.8

Substituting these results in eqn(3.3), gives a formula for the back scattered mode amplitude B_q. Therefore,

$$B_q = \frac{\int_{slot} (E_1 \times H_2).dS}{2 \int_{S_1} (E_{qt} \times H_{qt}).dS_1} \qquad 3.9$$

Repeating the same process, but with a change that (E_2, H_2) is assumed to be propagating in negative - Z direction, the equations for M_1 and M_2 are

$$M_2 = 2C_q \int_{S_2} (E_{qt} \times H_{qt}).dS_2 \qquad 3.10$$

and M_1 is equal to zero 3.11

Substituting the equations (3.10) and (3.11) in equn.(3.3), we get the formula for the forward-scattered mode amplitude C_q.

$$C_q = \frac{\int_{slot} (E_1 \times H_2).dS}{2\int_{S_2} (E_{qt} \times H_{qt}).dS_2} \qquad 3.12$$

From Fig[3.1], it is clear that the surfaces S_1 and S_2 in equns (3.9) & (3.12) are same. In these equations the denominators are equal, numerators need not necessarily be equal, because H_2 in equn.(3.9) is associated with a +Z propagating mode and H_2 in equn.(3.12) is associated with a -Z propagating mode.

3.3 GEOMETRY OF A SLOT FED BY A NRD GUIDE

Fig[3.1] shows the top view of a Slot cut in the upper ground plane of the NRD guide. The Slot is in the X-Z plane. It is centered with respect to the guide axis and is also inclined. If the Slot has an arbitrary inclination w.r.t the main axes, a separate co-ordinate system is considered for the Slot. The Slot axis is θ away from the NRD guide axis. Fig[3.2] shows various Slot parameters and its inclination with the guide axis. The length of the Slot is `2l', and the width is `W'. For a narrow Slot the width of the Slot must be very very small compared to its length.

As we have seen earlier a NRD guide, designed to propagate the PM_{11} mode, can also propagate PE_{11} mode and TE_{m0} modes. But, we can

design an excitation mechanism which excites only PM_{11} mode. Due to the presence of the Slot discontinuity, the power is scatter in all the higher order modes. The evanescent modes decay very fast away from the Slot in both +Z and -Z directions in the NRD guide. Therefore, if we consider the two planes, S_1 and S_2 of the Slot theory given in sec(3.2), sufficiently away from the Slot location, we can assume the fields to be consisting of only the propagating modes of the NRD guide i.e PM_{11}, PE_{11} and TE_{m0} modes. But, only TE_{m0} mode is considered though all TE_{m0} modes are propagating modes, because of the assumed Slot symmetry with respect to the NRD axis.

3.4 SCATTERING MATRIX FOR AN ISOLATED SLOT FED BY A NRD GUIDE

A scattering matrix is used in single moded transmission line systems, the elements of which give the transmission and reflection coefficients due to a discontinuity, that scatters the incident power. Now, in the case of a NRD guide, which is used at millimeter wave frequencies, for a given design, at least three modes propagate, neglecting the higher order TE_{m0} modes. So, the extension of the scattering matrix to multi-mode transmission line systems gives the Generalised Scattering Matrix (GSM). The GSM in general is of infinite order.

Let us consider that the NRD guide supports the PM_{11} dominant mode only. A conventional two-port scattering matrix can be used to represent the transmitted and scattered power due to the discontinuity. But, as stated in sec(3.3), we have to consider PE_{11} and TE_{10} modes, for a NRD guide designed to propagate PM_{11} mode. Each mode, can be represented by a two-port conventional scatter matrix. Therefore, the Slot discontinuity in the NRD guide, can be represented by a six-port

scatter matrix. If we take higher order modes, the order of the GSM increases. Fig[3.3], shows the scatter matrix for a Slot discontinuity in a NRD guide, supporting PM_{11}, TE_{10} and PE_{11} modes.

Using the Slot radiator theory given in sec(3.2), one can calculate the back-scattered and forward-scattered mode amplitudes for a single mode transmission system. Hence, using this theory the elements of the six-port matrix in Fig[3.3], can be determined. To determine the reflection co-efficient at port i, it is assumed that all the other ports are match terminated. Similarly, the transmission/coupling coefficients are determined. Therefore, calculation of B_q and C_q from equations (3.9) & (3.12) is essential for the determination of the scattering matrix elements. The unknown elements to be determined are considerably reduced if the network is reciprocal, loss less and passive.

3.5 SIGNIFICANCE OF BACK-SCATTERED(B_q) AND FORWARD-SCATTERED(C_q) MODE AMPLITUDES

The backward and forward scattered mode amplitudes, are not only used to calculate the elements of the scatter matrix S_{ij}, but also, provide information to determine the equivalent transmission line circuit for a single moded system.

The equations(3.9) & (3.12) show that the Slot does not scatter equally in both the directions. Moreover the Slot will couple to the p^{th} mode, if and only if it cuts the current lines corresponding to that mode. If the Slot parameters are small compared with the wavelength at operating frequency, the variation of the phase factor $\exp(\pm j\beta z)$ can be neglected and without loss of generality the Slot can be located at Z=0. There are two conditions for B_q and C_q:

i) $B_q = C_q$. In this case equn (3.1), shows that E_{1t} is continuous, at the plane Z=0, while the magnetic field H_{1t} is discontinuous at Z=0. With respect to the p^{th} mode the Slot discontinuity in the NRD guide reduces to a shunt element in the transmission line.

ii) $B_q = -C_q$. In this case E_{1t} is discontinuous and H_{1t} is continuous at the plane Z=0, as far as the p^{th} mode is concerned and the Slot discontinuity in the guide reduces to a series element in the transmission line.

But, in this thesis the Slots are those having a length of about $\lambda/2$, called resonant Slots and having a width small compared to the length. The electric field distribution in such a Slot is nearly sinusoidal along the length and independent of the feeding system. The direction of the field is transverse to the length of the Slot. In this case too, the Slot reduces to a series or shunt elements in the transmission line depending on the conditions:

iii) $B_q = C_q$ For this, the axis of the Slot is parallel to the guide axis. This condition results, not because of neglecting the variation of the phase factors $\exp(\pm j\beta z)$, but because the E_1 field in the aperture plane is an even function along the length of the Slot. Therefore, the Slot oriented in this manner behaves like a shunt element in equivalent transmission line representation.

iv) $B_q = -C_q$ Here, the axis of the Slot is perpendicular to the guide axis. In this case the transverse component of the E_1 field is zero, but the transverse magnetic field component H_2 is not zero. The variation of the phase factor $\exp(\pm j\beta z)$ can be replaced by unity. The Slot in the NRD guide is equivalent to a series element in transmission line representation.

From ,Fig[3.2], which shows the Slot geometry, it does not correspond to any of the special cases, discussed above. In this case, $B_q \neq C_q$ and the Slot behaves like a more complicated combination of series and shunt elements. In this case, the equivalent circuit representation of the Slot in the transmission line is either a T-section or Π-section.

3.6 DETERMINATION OF THE SCATTERING MATRIX ELEMENTS `S_{ij}'

An NRD guide designed to propagate only PM_{11} mode, also propagates PE_{11} and TE_{10} modes. This was stated earlier too. The scattering matrix required to represent a Slot discontinuity in such a guide is a six-port network. Now, to define the matrix elements fully, it is required to evaluate 36 coefficients. But, with the aid of the conditions of symmetry, reciprocity and passivity, the calculation of the number of element S_{ij} reduces drastically. The elements of the matrix, from Fig[3.3] are :

$$\begin{bmatrix} \left[S^{1}_{ij} \right] & | & | \\ | & \left[S^{2}_{ij} \right] & | \\ | & | & \left[S^{3}_{ij} \right] \end{bmatrix}$$

Where index notation represents :

$S^1_{ij} \quad = Bij/A_{11}, \quad i = 1,2$ and $j = 1,2$

$S^2_{ij} \quad = Bij/A_{22}, \quad i = 3,4$ and $j = 3,4$

$S^3_{ij} \quad = Bij/A_{33}, \quad i = 5,6$ and $j = 5,6$

NOTE: The dashed lines represent the other column elements. The j index for the columns shown by the 2x2 matrix are same as that of the dashed sub-matrix , whereas, the i index is from 1 to 6 excluding the the numbers shown by the i index in the 2x2 matrix. The even elements in each column of the 6x6 matrix has Bij = Cij.

The 2x2 matrices shown by the dashed lines denote the matrix of a single mode transmission system. The other elements of the 6x6 matrix are the mutual coupling coefficients between different modes. To evaluate the mutual coupling coefficient Bij or Cij (where i is the index for the row and j is the index for the column), the incident mode corresponds to the mode at the j^{th} port and the reflected field corresponds to the mode at the i^{th} port. For example, B_{52} denotes, that the incident mode amplitude is of PM_{11} mode and the field H_2 in the equn.(3.9) corresponds to TE_{10} mode. In determining the coefficients the incident mode amplitude for a particular mode at its two ports is taken to be same. Ports 1 & 2 denote PM_{11} mode, hence incident mode amplitude looking into the ports from either side is assumed to be equal. These coupling scatter coefficients give the amount of power coupled to the other modes. [APPENDIX II].

Aii, i=1,2,3 are the incident mode amplitudes of PM_{11}, PE_{11}, TE_{10} modes respectively. All the coefficients can be determined using eqns.(3.9) & (3.12). The scattering coefficients change with frequency. They remain constant for a very narrow band of frequencies. If all the higher order modes are taken into account, the order of the scatter matrix increases and a better accuracy is obtained. But, the

in the present thesis the order of the matrix is confined to include scattered fields due to 3 modes mentioned above.

3.7 DETERMINATION OF THE BACK AND FORWARD SCATTERED MODE AMPLITUDES, Bij & Cij.

The coefficients Bij & Cij can be determined considering that only one mode is incident at a time. In that the theory in sec(3.2) can be used to evaluate these coefficients.

i) B11 & C11 for PM_{11} MODE

B11 & C11 are the back-scattered and forward-scattered mode amplitudes respectively for PM_{11} incident mode. These coefficients can be determined as stated earlier from eqns.(3.9) & (3.12).

Elliot[14] points out that the best approximation to the field distribution in a Slot cut in the walls of any guide for any propagating mode is given by

$$E_1 = E_1(\zeta)\ \eta = V_m/W \cos(\pi\zeta/2l)\ \hat{\eta} \qquad 3.13$$

In the eqns (3.9) & (3.12) H_2, E_{qt}, H_{qt} correspond to the incident mode fields. The field expressions to these modes are given in table I, of chap 2. The field H_2 is the magnetic field perpendicular to the Slot field distribution E_1 and therefore it is given by the Z-component of the incident mode in region 1 of the NRD guide shown in the Fig[2.1].

$$H_2 = H_{z1} \exp(-jk_\zeta\zeta)\ \hat{z} \qquad 3.14$$

From the Fig[3.2] which shows the orientation of the Slot w.r.t the guide axis and various other Slot parameters, using transformations of co-ordinate systems, the relation between the co-ordinates of the guide axis and that of the Slot axis is

$$X = \sin\theta\ \zeta + \cos\theta\ \eta \qquad 3.15a$$

$$Z = \cos\theta\ \zeta + \sin\theta\ \eta \tag{3.15b}$$

$$k_\zeta = k_{x1}\ \cos\theta \tag{3.15c}$$

Substituting the eqns.(3.13), (3.14) & (3.15) in eqn.(3.9), the

$$\text{numerator} = \int_{-l}^{l} \int_{-W/2}^{W/2} E_1 \times H_2 \ . \ dS$$

$$= V_m/W \int_{-l}^{l} \int_{-W/2}^{W/2} H_{z1}\ \cos\theta\ \cos(\pi\zeta/2l)\ \exp(-jk_\zeta\zeta)\ d\eta\ d\zeta$$

$$= V_m/W\ C\ k_1^2 \cos\theta\ \pi/2l\ S_I\ T_2^+ \tag{3.16}$$

Where the different variables in the above equn. are

$$H_{z1} = C\ k_1^2\ \cos(k_{x1}x)\ \cos(\pi/b)y$$

$$S_I = \sin(k_{x1}\cos\theta\ W/2)/(k_{x1}\cos\theta)$$

$$T_2^+ = \cos(\beta_\Sigma l)/((\pi/2l)^2-\beta_\Sigma^2) + \cos(\beta_\Delta l)/((\pi/2l)^2-\beta_\Delta^2)$$

$$\beta_\Sigma = k_{x1}\ (\cos\theta + \sin\theta)$$

$$\beta_\Delta = k_{x1}\ (\sin\theta - \cos\theta)$$

The denominator in eqn.(3.9) is

$$T_{11} = \int_{S_1} E_{qt} \times H_{qt}\ .dS_1$$

$$= \int_{-a}^{a} \int_{0}^{b} E_{x1}\ H_{y1}\ dx\ dy + 2 \int_{-\infty}^{-a} \int_{0}^{b} E_{x3}\ H_{y3}\ dx\ dy$$

$$= GFb\ /\ 4k_{x1}\ (2k_{x1}a + \sin(2k_{x1}a)) +$$

$$R1\ R2\ (b/2k_{x2}) \tag{3.17}$$

where the constants are given by:

$$F = jC\beta\ k_1^2/(\pi/b)$$

$$G = jC\ (\omega\ \mu_o\pi/b + (\beta^2 k_{x1}^2)/(\omega\ \varepsilon_o\varepsilon_r\ \pi/b))$$

$$R1 = jC\ k_1^2/k_2^2(\pi/b\ \omega\ \mu_o\cos k_{x1}a - \beta^2 k_{x1}k_{x2}/\omega\ \varepsilon_o\varepsilon_r\pi/b\ \sin k_{x1}a)$$

$$R2 = jC\beta\ k_1^2/k_2^2(\pi/b\ \cos k_{x1}a - k_{x1}k_{x2}/(\varepsilon_r\pi/b)\ \sin k_{x1}a)$$

Therefore, the back-scattered mode amplitude for PM_{11} incident

mode B11, is obtained by substituting eqn. (3.17) & (3.16) in eqn.(3.9).

$$B11 = T_4 S_I \overset{+}{T}_2 \,/\, T_{11} \qquad 3.18$$

Where $T_4 = Vm/W \; Ck_1^2 \cos\theta \; \pi/2l$

Similarly following the same procedure C11, the forward-scattered mode amplitude for this mode can be calculated with the only change, that the field H_2 is propagating in the negative Z-direction. The denominator in the two equations (3.9) & (3.12) is however the same.

Therefore, the numerator of C11 is

$$= \int_{-1}^{1} \int_{-W/2}^{W/2} Vm/W \cos(\pi\zeta/2l) \; H_{z1} \exp(jk\,\zeta) \, d\eta \, d\zeta$$

$$= T_4 \, S_I \overset{+}{T}_2 \qquad 3.19$$

substituting eqn.(3.19) & (3.17) in eqn.(3.12),

$$C11 = T_4 S_I T_2 \,/\, T_{11}$$

Therefore from equations (3.18) & (3.20), the condition obtained is

$$\boxed{B11 = C11}$$

Therefore, the Slot in the NRD guide for a PM_{11} incident mode reduces to a shunt element in the transmission line representation. This also shows that the field in the aperture plane produces a discontinuity in modal current.

ii) B22 & C22 for PE_{11} incident mode

B22 & C22 are the backward and forward mode amplitudes respectively for a PE_{11} incident mode. Since PE_{11} mode is orthogonal to PM_{11} mode, the field distribution induced in the Slot E_1 due to this mode is same as that given in eqn.(3.13). But, it has an additional phase difference of 90^o which is accounted for by

introducing a +j factor. So, for PE_{11} mode incidence, the Slot E field distribution is given by

$$E_1 = jV_m/W \cos(\pi\zeta/2l)\ \hat{\eta} \qquad 3.21$$

H_2, the magnetic field perpendicular to the electric field distribution E_1 in the Slot for calculation of B22 is

$$H_2 = H_{x1} \exp(-jk_\zeta\zeta)\ \hat{x} + H_{z1} \exp(-jk_\zeta\zeta)\ \hat{z} \qquad 3.22$$

Where H_{x1}, H_{z1} and other variables are given by

$$H_{x1} = AK \cos(k_{x1}x)\cos(\pi/b)y$$

$$H_{z1} = Dk_1^2 \sin(k_{x1}x)\cos(\pi/b)y$$

$$K = \varepsilon_o\varepsilon_r\pi/b + (\beta^2 k_{x1}^2)/(\omega\ \mu_o\pi/b)$$

Therefore from eqn(3.9), the numerator of B22

$$= \int_{-l}^{l}\int_{-W/2}^{W/2} E_1 \times H_2\ .d\eta\ d\zeta\ \zeta$$

$$= AV_m/W\ \pi/2l\ S_I(\ K \sin\theta\ T_2^{+} \ \ (-D)k_1^2\cos\theta\ T_2^{-})$$

where, A = constant

$$T_2^{-} = \left(\cos(\beta_\Sigma l)/((\pi/2l)^2 - \beta_\Sigma^2\right) - \left[\cos(\beta_\Delta l)\backslash(\pi/2l)^2 - \beta_\Delta^2)\right]$$

The denominator in eqn.(3.9) is

$$T_{22} = \int_{-a}^{a}\int_{0}^{b} E_{qt} \times H_{qt}\ .z\ dx\ dy$$

$$= -\int_{-a}^{a}\int_{0}^{b} E_{y1}\ H_{x1}\ dx\ dy\ \ - 2\int_{-\infty}^{\infty}\int_{0}^{b} E_{y3}\ H_{x3}\ dx\ dy$$

$$= -MNb/4k_{x1}\ (2k_{x1}a + \sin(2k_{x1}a)) - \qquad 3.24$$

$$R1'\ R2'\ b/2k_{x2}$$

Where the constants M, N and other variables are:

$$M = jD\ \omega\ \mu o\ k_1^2/k_{x1}$$

$$N = jAK$$

$$R1' = jD\ k_1^2\ \omega\ \mu_o/k_{x1}\ \cos k_{x1}a$$

$$R2' = jA\ k_1^2/k_2^2\ (\omega\ \varepsilon_o\pi/b - \beta^2 k_{x2}^2/(\omega\ \mu_o\pi/b))\ \cos k_{x1}a$$

Substituting eqns.(3.23) & (3.24) in eqn.(3.9)

$$B22 = -V_m/W\ \pi/21\ S_I(\ AK\ \sin\theta\ T_2 - Dk_1^2\ \cos\theta\ T_2\ )/\ T_{22}$$

Following the same procedure to calculate C22, the forward-scatter mode amplitude due to PE_{11} mode, with the field H_2, propagating in the negative Z-direction.

Numerator of C22 = - (of the numerator B22) 3.26

The denominator of C22 is same as equation (3.24)

Finally, the expression for C22 can be obtained by substituting eqns.(3.24) & (3.26) in eqn.(3.12). The condition obtained for PE_{11} mode incidence is

$$\boxed{B22 = -C22}$$

This condition gives rise to a series element in transmission line equivalent circuit. This also shows that the Slot produces a discontinuity in modal voltage.

iii) B33 & C33 for TE_{10} mode

B33 & C33 are the back-scattered and forward-scattered mode amplitudes due to the TE_{10} incident mode. PE_{11} mode is a degenerate TE_{10} mode. So, the Slot field distribution due to the TE_{10} mode is same as that due to PE_{11} mode.Therefore, the field distribution in the Slot is

$$E_1 = jV_m/W\ \cos(\pi\zeta/21)\ \hat{\eta}$$

The magnetic field H_2 perpendicular to the field E_1 in the Slot is

$$H_2 = -jD/\beta\ H_{x1}\ \exp(-jk_\zeta\zeta)\ \hat{x} + H_{z1}\ \exp(-jk_\zeta\zeta)\ \hat{z} \qquad 3.27$$

Where H_{x1} and H_{z1} are the field components of TE_{10} mode in the NRD guide and are given in table III of chap 2.

Hence, the numerator of B33

$$= -DV_m/W\ \pi/21\ S_I\ (\ \beta k_{x1}\ \sin\theta\ T_2^{+} + k_1^2\ \cos\theta\ T_2^{-}\ ) \qquad 3.29$$

The denominator of B33,

$$T_{33} = \int\int E_{qt} \times H_{qt}\ .\ z\ dx\ dy$$

$$= -\int_{-a}^{a}\int_{0}^{b} E_{y1}\ H_{x1}\ dx\ dy\ -2\int_{-\infty}^{\infty}\int_{0}^{b} E_{y3}\ H_{x3}\ dx\ dy$$

$$= -LPb\ /\ 2k_{x1}\ (2k_{x1}a + \sin(2k_{x1}a))\ -$$
$$R1''\ R2''\ b/k_{x2} \qquad 3.30$$

where the constants L, P, $R1''$, $R2''$ are given in table III of chap2.

Therefore, substituting eqn.(3.29) & (3.30) in eqn.(3.9)

$$B33 = -DV_m/W\ S_I\pi/21\ (\beta k_{x1}\ \sin\theta\ T_2^{+} + k_1^2\cos\theta\ T_2^{-})\ /\ T_{33} \qquad 3.31$$

Following, similar procedure for determination of C33, But with a change in the direction of field H_2 the forward-scattered mode amplitude is

$$C33 = DV_m/W\ S_I\pi/21\ (\beta k_{x1}\ \sin\theta\ T_2^{+} + k_1^2\cos\theta\ T_2)/\ T_{33} \qquad 3.32$$

The expressions governing the back and forward scattered mode amplitudes give the relation :

$$\boxed{B33 = -C33}$$

The above relation leads us to the conclusion that the Slot discontinuity in the NRD guide is equivalent to a series element in transmission line representation,thereby producing a discontinuity in modal voltage.

3.8 CONCLUSION

It is shown in this chapter, that one of the simpler ways to analyze a Slot in a NRD guide is to represent it as an equivalent

scattering matrix. The 2-port transmission line equivalent circuit can also be used, but an NRD supports multi modes, therefore analysis of the Slot discontinuity in the guide becomes complicated, as each mode see's a different Slot impedance.

For a single PM_{11} mode propagation, the transmission line equivalent circuit for the Slot discontinuity in the guide is a shunt element. For a single mode propagation of PE_{11} and TE_{10} modes independently in the NRD guide, the Slot discontinuity is equivalent to a series element in the 2-port transmission line representation. Hence, for multi mode propagation (i.e PM_{11} ,PE_{11} ,TE_{10}), the Slot in the NRD guide is equivalent to a T or Π - network.

The plot of $|S_{11}|$ with frequency (the reflection coefficient at port 1 for PM_{11} mode) is practically insensitive to the variation in frequency for a bandwidth of 10Ghz. Fig[3.5] shows that the variation of normalised $|S_{11}|$ with 'θ', the inclination of the Slot with the guide center line. From this knowledge we can deduce that a longitudinal Slot, along the guide center line couples maximum power for PM_{11} mode and minimum for PE_{11} and TE_{10} incident modes. A Slot orthogonal to the guide center line, couples maximum power in case of PE_{11} and TE_{10} modes and the perturbation of the PM_{11} current lines is minimum.

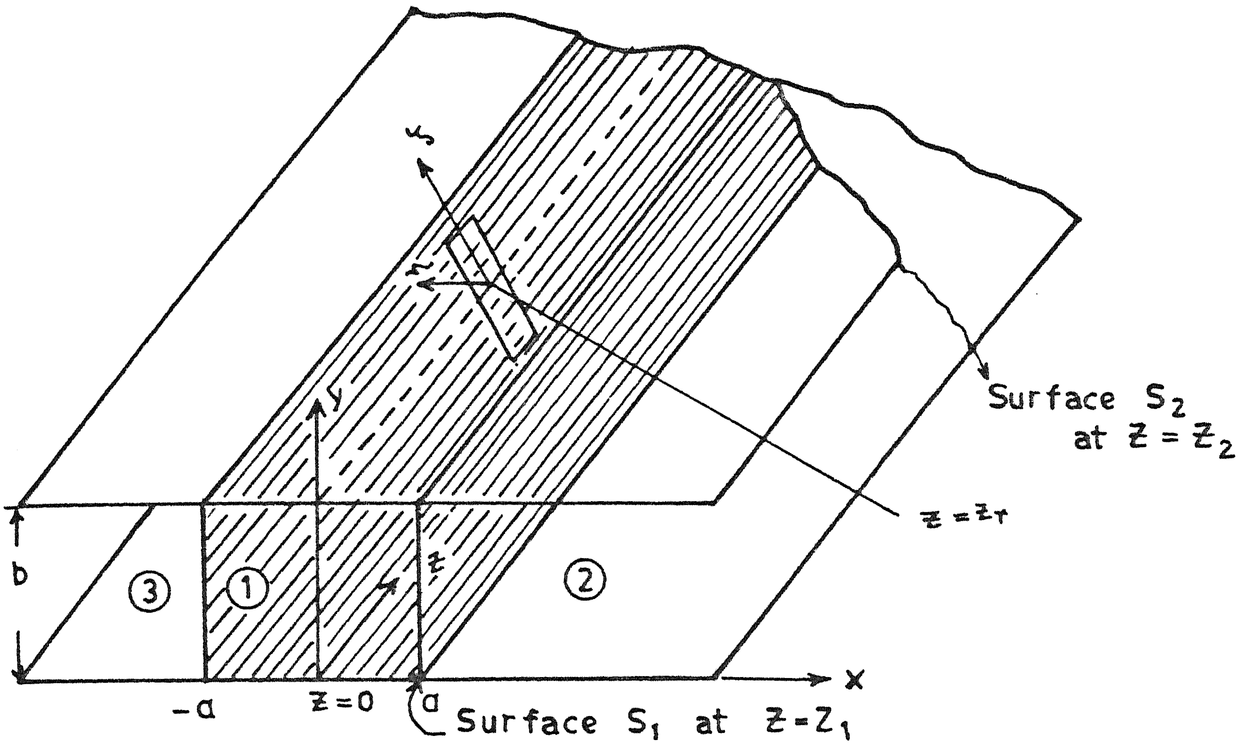


FIG.[3.1] Single slot fed by NRD guide.

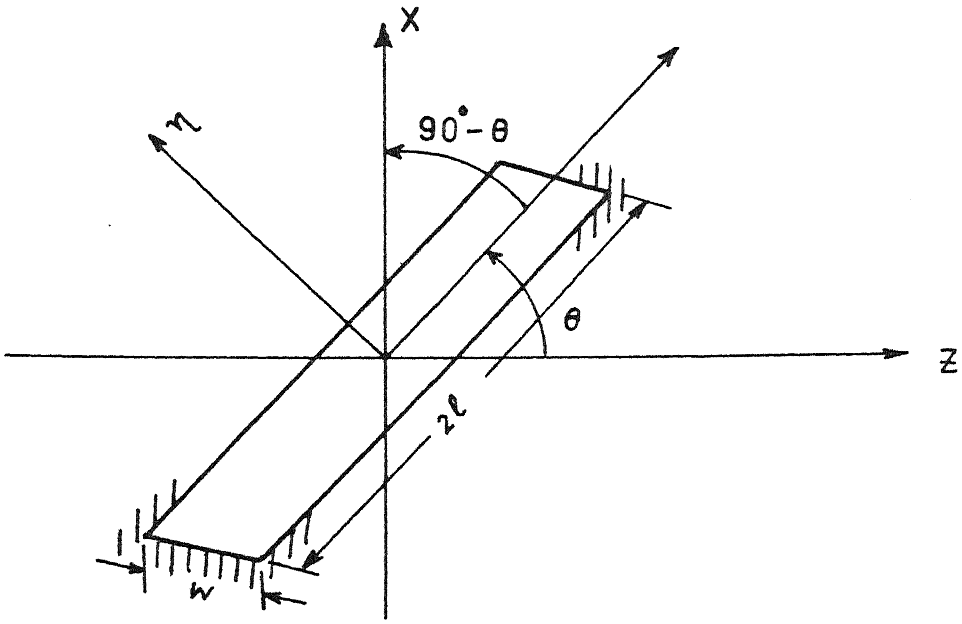


FIG.[3.2] Cross section of the slot defining the slot parameters.

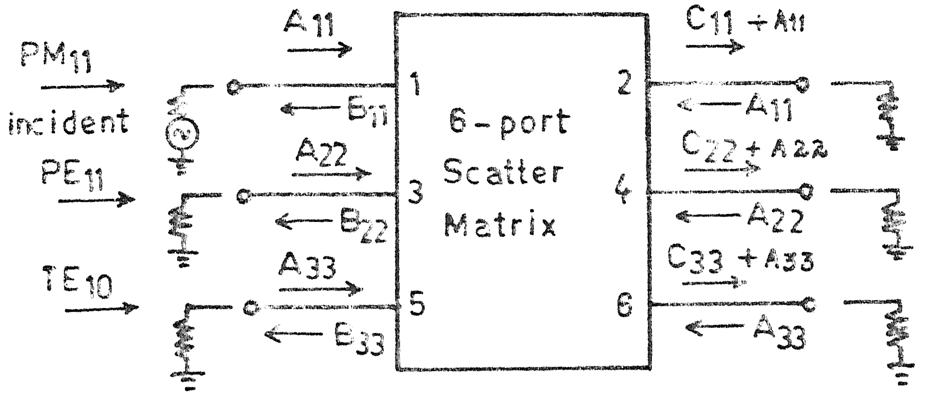
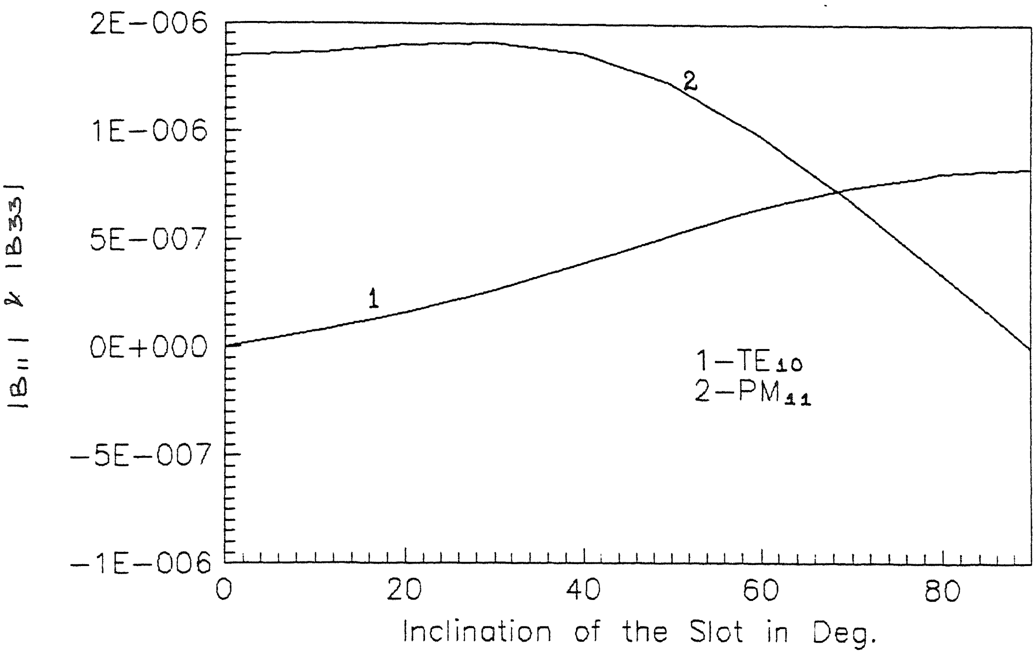
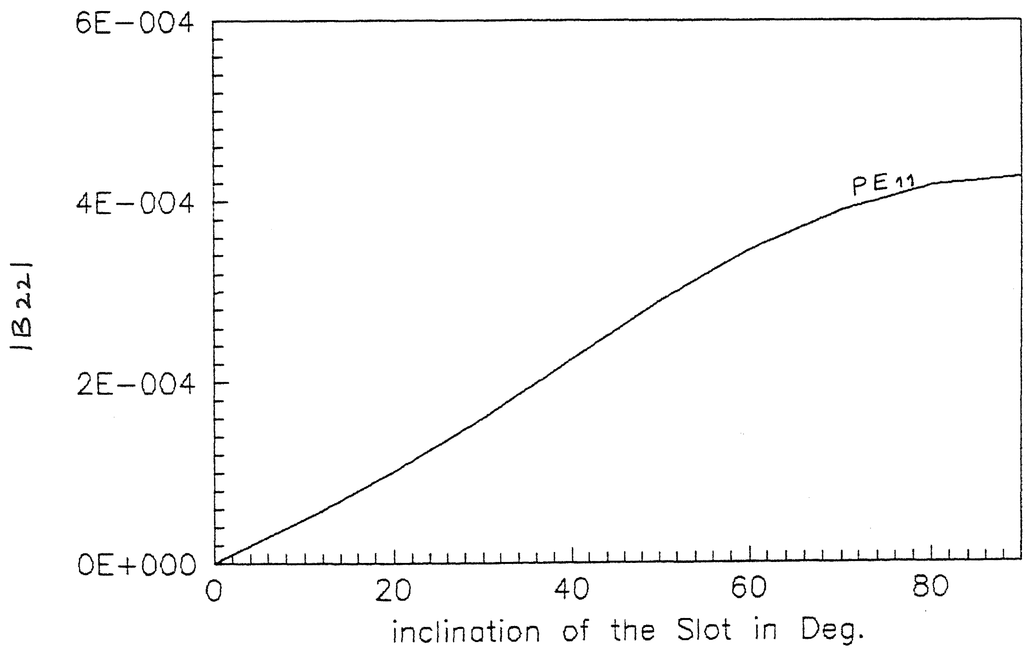


FIG.[3.3] Generalised Scattering Matrix for an Isolated Slot
(Excluding the Radiation Ports)



Fig[3·5a] Normalised Bii Vs Slot orientation. , i = 1 for PM_{11}
= 3 for TE_{10}

(ϵ_r = 2·13, b/a = 1·706, f = 95 GHz W = ·002m)



Fig[3·5b] Normalised Bii Vs Slot orientation., i = 2

(ϵ_r = 2·13 , b/a = 1·706 , f = 95 GHz , W = ·002m)

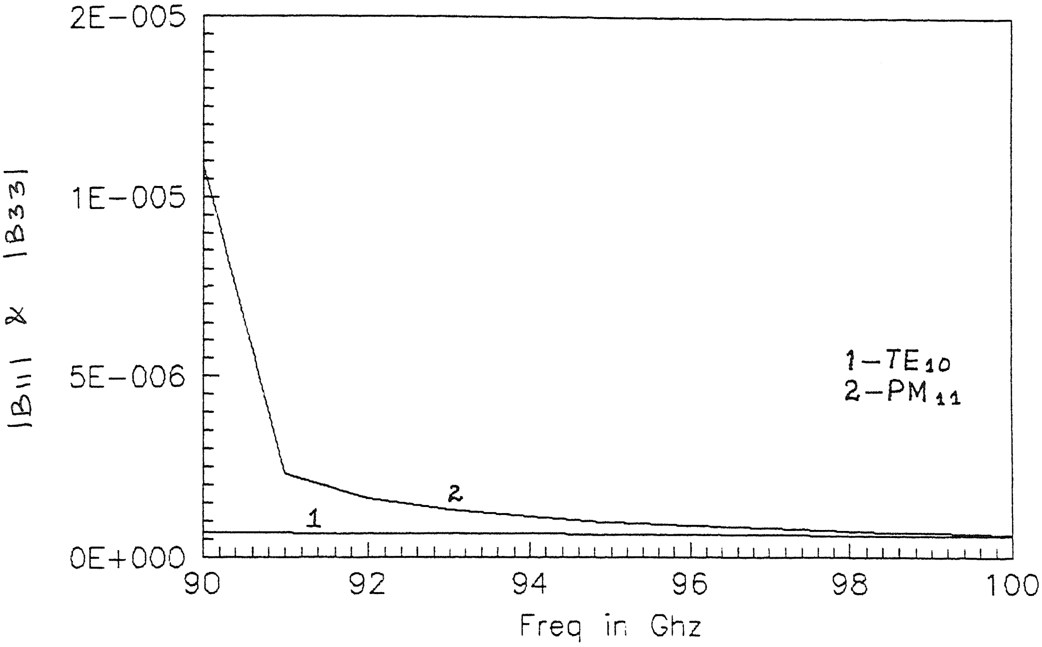


Fig 3·4a variation of Bii with freq., i = 1 for PM_{11}
= 3 for TE_{10}

(ϵ_r = 2·13, b/a = 1·706, $\theta = 60°$, W = ·002 m)

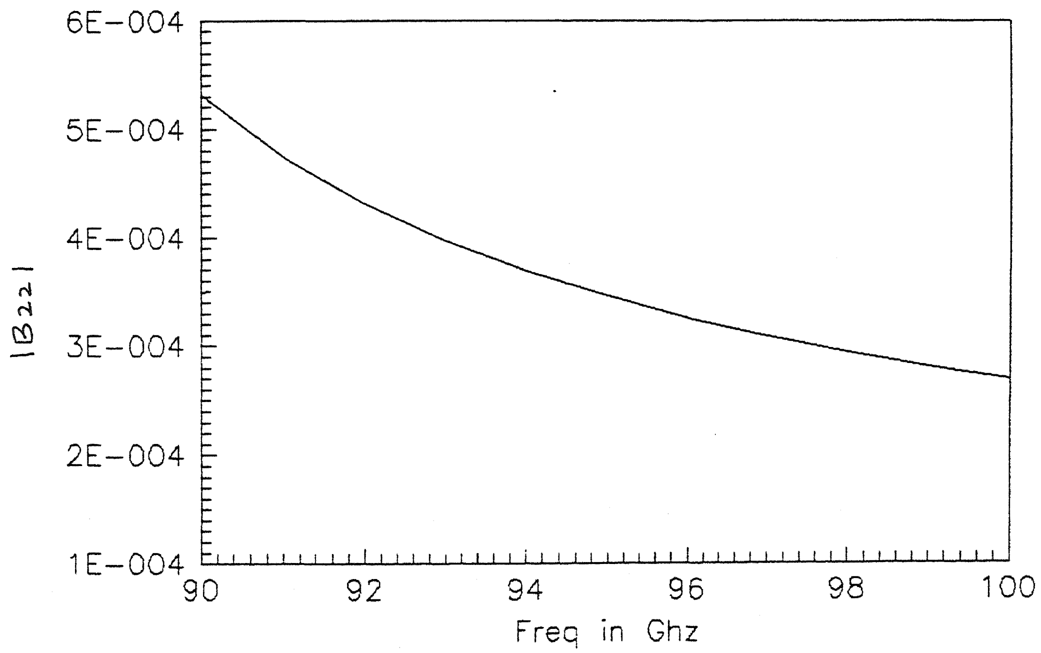


Fig 3·4b variation of Bii with freq for PE_{11} mode., i = 2

(ϵ_r = 2·13, b/a = 1·706, $\theta = 60°$, W = ·002 m)

CHAPTER 4

MUTUAL COUPLING BETWEEN SLOTS

4.1 INTRODUCTION

In this chapter, the mutual coupling between two Slots is analysed. For a Slot to be used in an array, it is necessary to determine the mutual impedance/admittance between the Slots which is due to the electromagnetic coupling between them. In a waveguide fed Slot array, the active admittance of each Slot can be defined in terms of the propagating waveguide mode incident on the Slot in conjunction with the propagating mode back-scattered by the Slot. But the sum of these oppositely travelling modes is not so easily linked to the electric field distribution in the Slot. One must determine this linkage in order to design waveguide fed arrays, and it must include the effects of mutual coupling between the Slots.

As a result of mutual coupling, the input impedance of the Slot changes. Therefore, the input impedance of Slots in an array is different from the input impedance of the isolated Slot. In the design of a linear array of Slots fed by NRD guide, it is desirable to adjust the power radiated by each Slot, according to some array excitation coefficients which maximise the gain or minimise the beam width for a given side lobe level. In the study of single slot it was shown that power coupled to a Slot depends on the orientation of the Slot w.r.t the NRD axis. Therefore by adjusting the angle "θ" we can control the amplitude coefficient and by properly choosing the spacings we can adjust the phase of the excitation to obtain the desired radiation

pattern.

therefore study of mutual coupling between two Slots as a function of position and orientation becomes important in the design of Slot array. Further, the problem is little more complicated in the case of NRD fed guide because it can propagate more than one mode for given dimensions of the guide. Although the excitation of the waveguide is confined to PM_{11} mode, the Slot discontinuity excites all other modes propagating as well as evanescent. The evanescent mode fields contribute to the reactive component of the in put impedance and are confined to the vicinity of the Slot. But the power scattered in the propagating modes PM_{11},$PE_{11}$and TE_{m0} modes propagate to other Slots, in turn, exciting the Slots with these mode fields. In order to design an array of Slots it is essential to determine the exact nature of the mode interaction.

so, in this chapter, the scatter matrix of a single Slot developed in the previous chapter is extended to a pair of mutually coupled Slots. A possible method of extension of the generalised scatter matrix representation to a multiple Slot geometry is included.

4.2 LOSSLESS SCATTERING MATRIX FOR A SINGLE SLOT FED BY A NRD GUIDE

A six port scatter matrix for a single Slot cut in the upper conducting plate of the NRD guide is already developed in chap3. The matrix takes into account the three propagating modes, PM_{11},PE_{11} and TE_{10}. The Slot discontinuity generates all the other modes too. For, greater accuracy, all the excited modes are to be considered and the scatter matrix extends to infinite order. The Slot discontinuity not only scatters the power in forward and backward directions but also radiates a part of it.

To determine the incident mode amplitude or the maximum voltage at the center of the Slot, it is essential to write a power balance equation. Since, the back and forward scatter coefficients and the power radiated into the half space in calculable, for each mode, the power balance eqn. leads us to the calculation of the incident mode amplitude and the maximum voltage "Vm" at the center of the Slot. To include the power radiated by the Slot, the six -port scatter matrix has to be extended to order nine. The power radiated by the Slot can be represented by a single port(7^{th}). But, each excited mode see's a different Slot impedance. Hence, the total power radiated is the sum of the individual power radiated by each excited mode.So, the power radiated by the Slot is represented by the 7^{th},8^{th} and 9^{th} ports corresponding to PM_{11},PE_{11} and TE_{10} mode excitation respectively. Further the radiation ports are not mutually coupled. They are isolated

with respect to each other. the sum of the reflection coefficients at these three ports is the reflection coefficient of the total radiation by the Slot. With the inclusion of these ports, the generalised scatter matrix is loss less, apart from being passive. The term loss less refers to multi ports free from internal losses of electromagnetic energy. This requires that the sum of powers entering a multi port network should be equal to the sum of powers leaving the network.

In reality, absolutely loss less microwave junctions do not exist;any circuit dissipates some of the power it receives. However, in many cases , every effort is made to minimise the internal losses. The low level losses should be understood as vanishingly small compared with the total power applied to an n-port. For a loss less and passive

multi port, the scattering matrix is such that

$$\left[S^*\right]^T\left[S\right] = I \qquad 4.1$$

Where $\left[S^*\right]^T$ is the complex conjugate transpose of the scatter matrix $\left[S\right]$ and I is the identity square matrix of order n. Eqn.(4.1) defines the unitary property in matrix algebra. The properties of the unitary matrix are (i) the sum of the squares of the magnitudes of the column elements is unity, (ii) the columns are mutually orthogonal and (iii) the determinant of a unitary matrix has a magnitude of unity. Using these properties, the elements of the scatter matrix, of the Slot resulting due to the inclusion of the radiation ports can be determined.

4.3 GEOMETRY OF TWO SLOTS FED BY A NRD GUIDE

Fig[4-1] shows the various parameters and the geometry of two Slots fed by the NRD guide. (η_1,ξ_1) and (η_2,ξ_2) are the axis of the Slots (1) and (2) respectively. The Slots are centered with respect to the guide center line and are alternately inclined. The length of slot(1) and that of Slot(2) is $2l_1$. Let θ_1 and θ_2 be the inclinations of the two Slots with respect to the guide center line. The distance of separation between the centers of the two Slots is `d'. Offsetting the Slots from the guide center line normally does not affect the polarisation characteristics obtained due to PM_{11} mode, because of the absence of the Hx component in its field. The magnitudes R_1 and R_2 denote the distances from the positive and negative ends of Slot(1) to that of Slot(2). R is the distance from any point on Slot(1) to that of any point on Slot(2). The distances defined in Fig[4-1] are given by

$$R^2 = \eta_2^2 + (\zeta_2-\zeta_1)^2 \qquad 4.2a$$

$$R_1^2 = \eta_2^2 + (\zeta_2-1)^2 \qquad 4.2b$$

$$R_2^2 = \eta_2^2 + (\zeta_2+1)^2 \qquad 4.2c$$

The adjacent Slot variables are related through [appendix I]

$$\eta_1 = d \sin\theta_1 + \zeta_2 \sin(\theta_2 - \theta_1) \qquad 4.3a$$

$$\zeta_1 = d \cos\theta_1 - \zeta_2 \cos(\theta_2 - \theta_1) \qquad 4.3b$$

The distance 'd' between the centers of the two Slots is chosen to be $\lambda_o/2$, where λ_o is the free space wavelength at the operating frequency. From Fig[2.7], it is evident that the guide wavelength is greater than the operating wavelength for the propagating PM_{11} and PE_{11} modes.So, $d \neq \lambda_g/2$ and the antenna is said to be travelling wave fed. The assumption of longitudinal electric field distribution in the Slot is however valid, because W << 21.

4.4 ANALYSIS OF COUPLING BETWEEN TWO SLOTS

The external mutual coupling between the Slots is analysed with the aid of reciprocity theorem. The approach followed is same as that given in ref[8].

The derivation of reciprocity theorem is based on the idea of two sets of sources (J^q, J_m^q, ρ^q, ρ_m^q) or (J^p J_m^p ρ^p ρ_m^p), can be established in a region producing the fields (E^q, H^q) and (E^p, H^p) respectively. J and J_m are bound current densities, which combine to give the total current density in the medium. It is assumed that the two sets of sources oscillate at a common frequency and the electromagnetic behaviour of the materials must be linear. For such a system, the reciprocity theorem takes the general form :

$$\int_S (E^q x H^p - E^p x H^q).dS = \int_V (E^p.J^q - \mu_o H^p.J_m^q - E^q.J^p + \mu_o H^q.J_m^p)\, dV \qquad 4.4$$

and the closed surface S encloses the volume V.[14].

4.5 METHOD OF ANALYSIS

Consider the two Slots shown in Fig[4·1]. Slot(2) for the time being is eternal to the guide and Slot(1) be situated at a position Z_0. If the NRD guide is assumed to be in finitely long or terminated and an NRD mode of amplitude A^q be incident on the Slot from -Z direction. The incidence of this mode on the Slot will cause forward and backward scattering of the incident mode and because of the Slot discontinuity other modes are generated in the vicinity of the Slot. In addition to this, radiation into outer space, via the electric field set up in the Slot takes place. Consider two situations p and q to derive an expression for the forward and backward scattering coefficient of the incident mode in the presence of the Slot(2) with the help of eqn(4.4).

Situation "q":

In this situation only Slot(1) is present and Slot(2) is closed by a conducting material. As a result of the incident mode field, a voltage $V_1^{s,q}$ develops at the center of Slot(1), and waves B^q and C^q are scattered from the Slot in the backward and forward directions respectively. The incident mode is replaced by equivalent electric and magnetic current sources K^q and K_2^q respectively and the reciprocity theorem is applied.

$$K^q = \hat{z} \times H^q$$

$$K_2^q = -\mu_o^{-1}\, \hat{z} \times E^q \qquad 4.5$$

Situation "p":

In this situation there is no incident mode field from the -z direction. Slot(1) is present in this case also, but along with sources of magnetic current sheets placed against the ground plane on the outer half space. These magnetic current sources will be so

chosen, as to contribute to the external field precisely as would the actual electric fields in the actual Slots. They will excite Slot(1) externally, the result being a mode wave of amplitude B^b propagates in the -z direction in the guide containing the Slot(1).

Applying the reciprocity theorem, in the form given in eqn(4.4), for this pair of situation we h ve

$$\int_{S_1} (E^q x\ H^p - E^p x\ H^q).\ dS_1 \ = \ \mu_o \int_{S_{(2)}} H^q_{ext} \cdot K^p_{2\ ext} \cdot dS_{(2)} \qquad 4.6$$

where S_1 is the surface representing the cross-section of the NRD guide and the surface $S_{(2)}$ is the cross-section of Slot(2).

The scattered field of Slot(1), resulting because of situation `p', can be represented as

$$H^p \quad = -\Sigma_p B^p\ H_{mnt} \exp(\gamma_{mn} z) \qquad z<z_1 \qquad 4.7a$$

$$E^p \quad = \ \Sigma_p B^p\ E_{mnt} \exp(\gamma_{mn} z) \qquad z<z_1 \qquad 4.7b$$

where z_1 denotes the front end of the module containing the slot. E^q,H^q are the of the incident mode whose amplitude is A^q. Therefore, fromeqn.(4.6)

$$\text{L.H.S} = \int_{S_1} A^q E_{mn} \exp(-\gamma_{mn} z_1) x - B^p H_{mn} \exp(\gamma_{mn} z_1).dS_1$$

$$- \int_{S_1} B^p E_{mnt} \exp(\gamma_{mn} z_1) x\ A^q H_{mnt} \exp(-\gamma_{mn} z_1).dS_1$$

$$= -2A^q B^p \int_{S_1} E_{mnt} x\ H_{mnt} .dS_1$$

$$= -A^{qi} B^p\ T_{ii} \qquad\qquad (A^{qi} = A^q) \qquad\qquad 4.8$$

m,n are the half sinusoidal variations in X & Y directions. As already stated the electric field distribution in Slot(1) is assumed to be

$$E^{s,a}_1 \qquad = V^{s,q}_1 / W \cos(\pi\zeta_1 / 2l)\ \eta_1 \qquad 4.9$$

The Slot parameters in the above eqn. are described in chap 3.The fields produced by this Slot i.e.Slot(1) in the half free space are

same as those from the magnetic current sheet

$$K_2^q = 2\mu_o^{-1} V_1^{s,q}/W \cos(\pi\zeta_1/2l)\ \hat{\zeta}_1 \tag{4.10}$$

The electric vector potential associated with this equivalent current sheet is

$$F(\xi,\eta,\zeta)_1 = 1/(4\pi\mu_o^{-1}) \int_{-1}^{1}\int_{-W/2}^{W/2} K_m^q/R \exp(-jk_oR)\ d\eta_1' d\zeta_1'$$

$$= V_1^{s,q}/2\pi \int_{-1}^{1} \cos(\pi\zeta_1'/2l) \exp(-jk_oR)\ /R\ d\zeta_1'\zeta_1$$

The magnetic field and vector potential are related by

$$H_{ext}^q = 1/(j\omega\mu_o)\ (k_o^2 F + \nabla(\nabla.F))$$

$$= 1/(j\omega\mu_o) \left| \frac{\partial^2}{\partial\eta_1\partial\zeta_1} \hat{\eta}_1 + \left(\frac{\partial}{\partial^2\zeta_1} + k_o^2\right) \hat{\zeta}_1 \right| F_{\zeta_1}$$

The magnetic current sheet in the position of slot(2) on surface S_2 is given by

$$K_{2ext}^p = \mu_o^{-1} V_2^{s,p}/W \cos(\pi\zeta_2/2l) \tag{4.13}$$

where $V_2^{s,p}$ is the voltage at the center of slot(2). Therefore, the backscattered wa v e coefficient due to external mutual coulig,

$$B^p = -1/A^{qi} T_{ii} \int_{-W/2}^{W/2}\int_{-1}^{1} \mu_o H_{ext}^q \cdot K_{2\ ext}^p\ d\zeta_2'\ d\eta_2' \tag{4.14}$$

$$i = 1,2,3$$

4.6 RELATION BETWEEN BACK SCATTERED WAVE AND ISOLATED SLOT ADMITTANCE FOR A SINGLE INCIDENT MODE

For a guide which excites a single mode and having a Slot discontinuity, the transmission line theory provides the equivalent circuit representation for the Slot depending on the condition of the scattered mode amplitudes and this has been analysed in chap 3.

For symmetrical back-scatter i.e B = C

$$Y_n/G_o = -\ 2B^q/(A^q+B^q) \tag{4.15}$$

Where Y_n is the Slot self admittance for that particular incident mode and G_o, the characteristic conductance of the equivalent transmission line. Solving eqn(4.15), an expression for A^q in terms of normalised Slot self admittance is obtained. Substituting
it in eqn(4.14) gives the relation between the back-scattered wave due to the mutual coupling and the isolated Slot admittance.

4.7 DEVELOPMENT OF SCATTER MATRICES TO ACCOUNT FOR MUTUAL COUPLING (EXTERNAL)

In sec(4.2), ports 7,8,9 represent the radiation of power on account of incident PM_{11}, PE_{11} and TE_{10} modes respectively. These three ports are isolated with each other. Since, it is assumed that the generalised scatter matrix of the slot is reciprocal and passive, the external mutual coupling can be accounted by developing another six-port scatter matrix.

The 7^{th},$8^{th}$and 9^{th} ports are the input ports to the mutual coupling matrix and the output ports are connected to the corresponding radiation ports of the next Slot matrix. A wave incident at one of the ports couples power to other modes. The scatter coefficients of this matrix can be determined in the same way as those developed for the isolated Slot matrix, but using the eqn.(4.14). For PM_{11} mode, the scattering is symmetrical (i.e B=C), whereas for PE_{11} mode and T_{10} mode, the scattering is anti-symmetrical (i.e B=-C).

To determine, the coupling between PM_{11} and PE_{11} mode, the back scattered wave is normalised with the incident mode under consideration and in eqn.(4.14) K^p_{2ext} is that due to the coupled mode.

4.8 DETERMINATION OF V_m

V_m is the maximum amplitude of the electric field distribution at the center of the Slot. Fig[4.2] shows the scattering matrix of two slots whose geometry is given in Fig[4.1]. In the previous section, eqn.(4.14) is used to calculate the elements of the mutual coupling matrix. The unknown variables in the eqn. are V_m and A^q, the incident mode amplitude.The incident power is assumed to be unity for the determination of the scatter coefficients, thereby calculating the the incident mode amplitude.

To calculate V_m, a power balance relation is written that will connect the Slot's excitation to its inclination with the guide center line. as already stated, power balance relation is possible because, the power contained in the incident mode, the power radiated into free space and the back and forward scatter mode amplitudes are all calculable and are given in chap 3. Each port of the multi-port scatter matrix shown in Fig[4.2] is excited one at a time in turn , while terminating all the other ports in matched loads. V_m for the three propagating modes can be calculated from the power balance relation.

Consider PM_{11} mode to be the incident mode and propagating in the +z-direction. Except port1 all the other ports are match terminated in their respective matched loads. Port 7, represents the power radiated due to PM_{11} excitation. Hence for port 1 excitation, port 7 of Slot(1) is only excited as far as radiation ports are concerned. V_{m1} is the maximum electric field distribution at the center of the Slot due PM_{11} mode excitation. V_{m2} , V_{m3} denote the same for PE_{11} and TE_{10} mode excitation respectively.

The power balance relation states that incident power is equal

to the sum of power reflected , power transmitted and power radiated.i.e

$$P_{inc} = P_{trans}+P_{refl}+P_{rad} \qquad 4.16$$

the power radiated by the Slot is calculated using the poynting vector theorem. The electric field in the Slot is assumed to be

$$E = V_{mi}/W \cos(\pi\zeta/2l)\,\hat{\eta} \qquad 4.17$$

$$i=1,2,3$$

All the parameters in eqn(4.17) are already explained in chap 3.

$$E = j\omega\ \hat{r}\ x(\hat{r}xA)$$

$$H = B/\mu_o = -j\omega\ /\eta\ \ \hat{r}\ x\ A$$

$$= 1/\eta\ \ \hat{r}\ x\ E$$

where $\eta = \mu/\varepsilon$ and $\hat{r}$ is the unit vector in the radial direction.

Therefore using eqn(4.17)

$$H = V_{mi}/W\eta \cos(\pi\zeta/2l)\ \hat{\zeta} \qquad 4.18$$

Using poynting theorem,

$$P_{rad} = 1/2\ Re\int_{slot} E\ x\ H^* \ .\ dS$$

$$= |V_{mi}|^2\ (1/2\eta W) \qquad 4.19$$

Similarly using the poynting vector theorem, incident power is

$$P_{inc} = 1/2\ Re\int_{S_1} (A^{qi}E\ x\ A^{qi*}H^*)\ .dS_1$$

$$= 1/2\ T_{ii}|A^{qi}|^2 \qquad ,i=1,2,3 \qquad 4.20$$

The subscript i stands for the incident mode fields, PM_{11} , PE_{11} and TE_{10} respectively. For i=1, $T_{11}$is the power in the guide due to PM_{11} mode and A^{q1} is the incident mode amplitude of the mode. Surface S_1 is the cross-section of the NRD guide. The reflected power,

$$P_{refl} = 1/2\ T_{ii}|B^{qi}|^2 \qquad 4.21$$

and the transmitted power is

$$P_{trans} = 1/2\ T_{ii}|A^{qi} + C^{qi}|^2 \qquad 4.22$$

Substituting eqns.(4.19) to (4.22) in eqn.(4.16) gives the relation to determine the variable V_{mi}. Once V_{mi} is determined for the three propagating modes, the elements of the mutual coupling scatter matrix can be determined, using the approach given in sec(4.7).

4.9 DEVELOPMENT OF MATRICES TO ACCOUNT FOR INTERNAL MUTUAL COUPLING

The radiation ports of Slot(1) and that of Slot(2) are externally connected by the six-port mutual coupling scatter matrix. Fig[4·2] shows the individual multi-ports of Slot(1) and that of Slot(2) are internally connected by transmission lines.

From Fig[4·1], ports 2,4,6 of Slot(1) are cascaded with ports $1', 3', 5'$ of Slot(2). the transmission line, connecting port 2 and port $1'$ has the characteristic impedance and propagation constant of the propagating mode, PM_{11}. Therefore, if port 1 is excited with all the

other ports match terminated , only port $1'$ excited internally.Port 4 connects port $3'$ for PE_{11} mode excitation, internally.finally, port 5 connects port $6'$ for TE_{10} mode excitation.

The characteristic impedance of the transmission lines, for the three different propagating modes and the corresponding propagation constants are determined from the characteristic eqns. given in chap 2.

4.10 EXTENSION TO N-SLOT LINEAR ARRAY

In this section, equations resulting as result of extending the NRD fed Slot N-Slot linear array are developed. The problem extends to the interconnection of the individual scattering matrices and the

mutual co   atter matrix. In sec(4.5), the expression for back scat---- ---e amplitude, due to the external mutual coupling is obtained for two Slots.

For a linear array of N-Slots alternately inclined and equally spaced, the eqn.(4.14) giving the expression for back scattered wave due external mutual coupling modifies to

$$B^{pi} = -1/(A^{qi}\ T_i) \sum_{m=2}^{N} \int_{-W/2}^{W/2}\int_{-1}^{1} \mu_0 H^q_{ext} \cdot K^p_{m1\ ext}\, d\zeta'_m\, d\eta'_m \qquad 4.23$$

In case of N-Slots, the situations `p' and `q' of sec(4.5) are also modified which results in the eqn(4.23). In situation `p', there are N-Slots, (N-1) of which are for the time being external to the guide. In `q' situation , except the n^{th} Slot all the other (N-1) Slots are closed by conducting material. The procedure followed in sec(4.5), with the only exception that there are N-Slots instead of two Slots.

The radiation ports(7,8,9) are interconnected with the corresponding radiation ports of the remaining (N-1) Slot equivalent network via a scattering matrix to take into account the external mutual coupling. For, an increase of one Slot in the linear array three additional ports are added to the external mutual coupling matrix, because only three modes are taken into consideration. Fig[4.3] shows the scatter matrix representation of an n-element linear array. Appendix I gives the relation between the n^{th} and m^{th} Slot variables.

4.11 CONCLUSION

A nine port Scattering Matrix is needed to represent the Slot discontinuity in the NRD waveguide. The Matrix is lossless, apart from being passive and reciprocal. Appendix III gives the Scattering Matrix

elements of a single Slot, calculated at an operating frequency 95Ghz with Slot oriented at an angle 60° with respect to the guide center line and NRD dimensions are calculated in sec(2.6).

To account for external mutual coupling, a mutual coupling matrix of order six is needed for two Slots. For a N-element array the mutual coupling matrix order extends to 3N. The elements of this Scattering matrix can be determined solving the eqn.(4.14) and using eqns.(4.2) & (4.3). After integration B^{p} for the three modes under consideration is calculable because, T_{ii} and A^{qi} are known for each of the modes. Once B^{p} is calculated, the determination of the mutual coupling matrix is trivial.

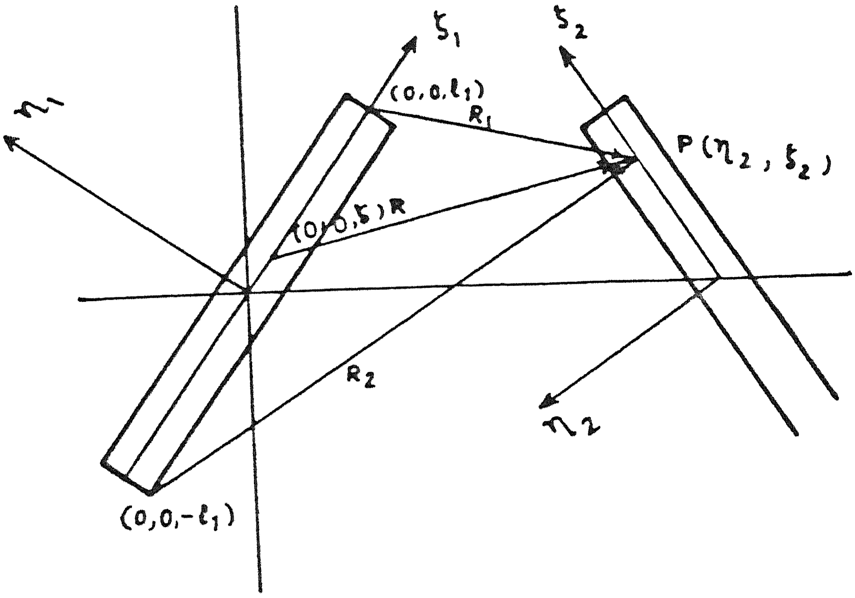


FIG. [4·1] Geometry of two slots.

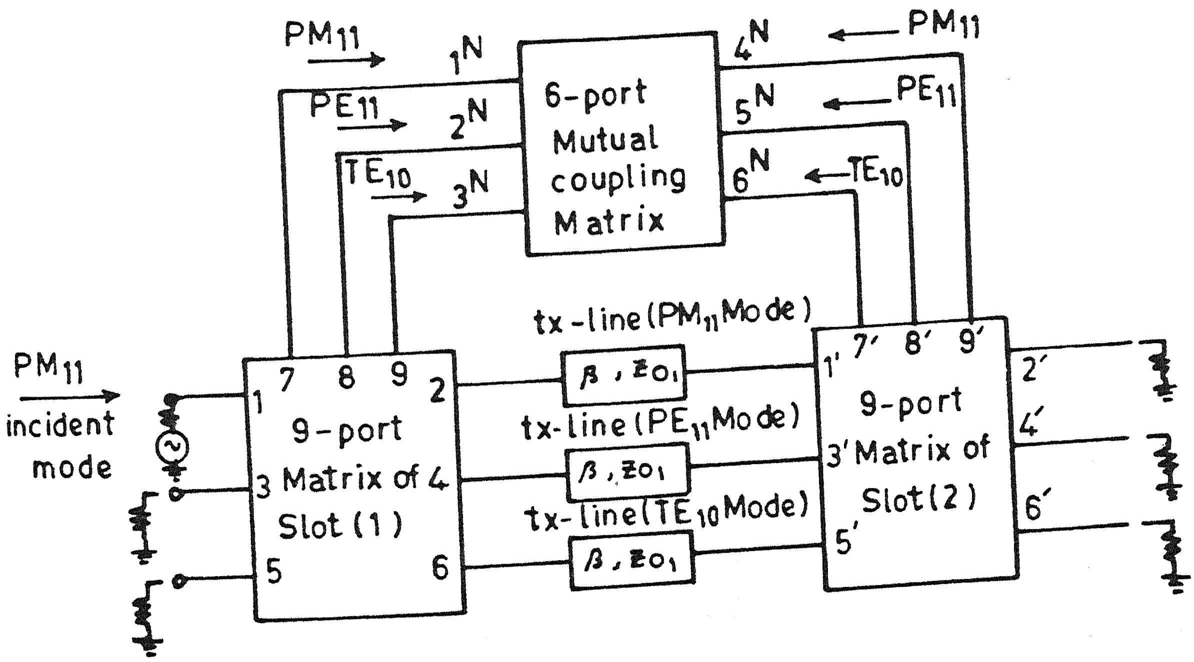


FIG. [4·2] Generalised Scattering Matrix for 2-slots.

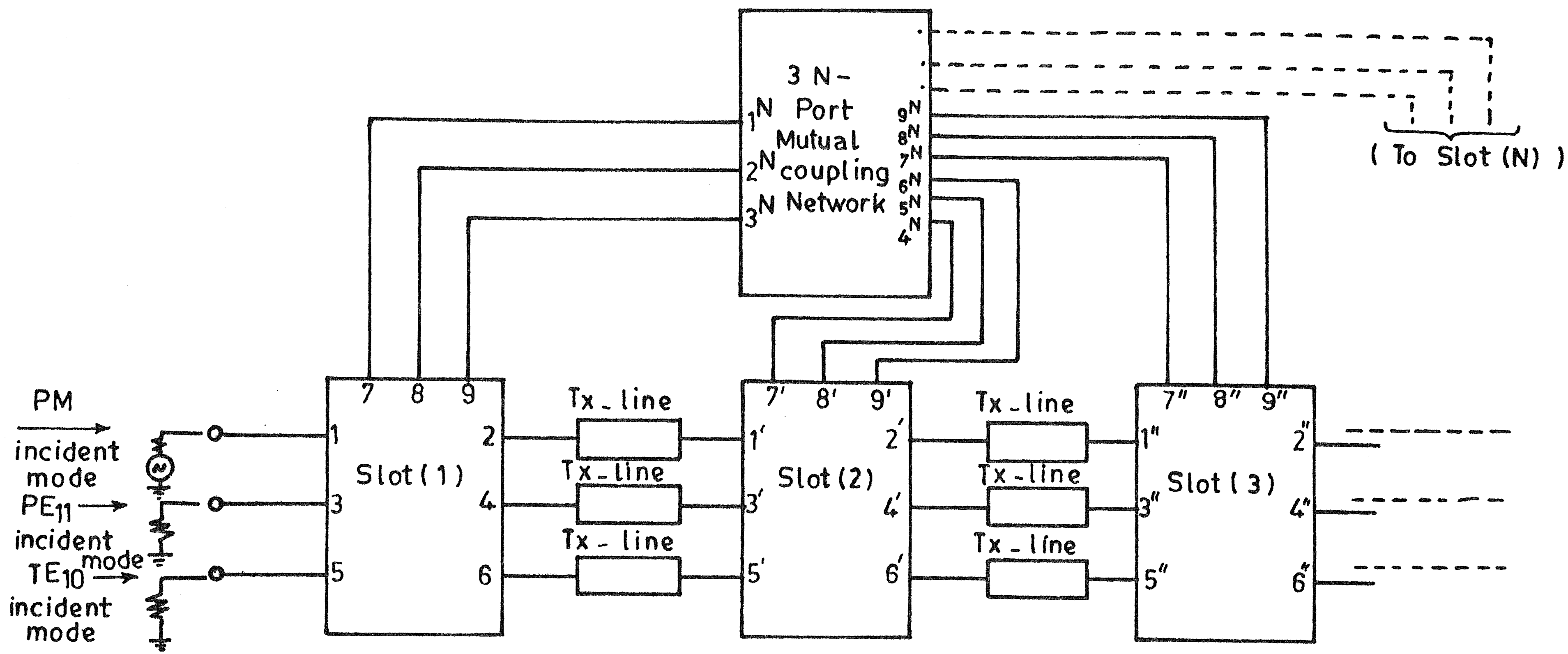


FIG.[4·3] Generalised Scattering Matrix of N-Element linear slot array.

CHAPTER 5

SUMMARY AND CONCLUSIONS

5.1 SUMMARY AND CONCLUSIONS

The NRD guide introduced by Yoneyama and Nishida and its development later is a useful medium to realize millimeter wave integrated systems.To obtain the full benefits of an integrated system,an antenna in the same system should be designed to avoid costly transitions, both from performance wise and economic point of view.

A Slot array fed by a NRD guide is one such antenna, which has promising usage at millimeter wave frequencies. The NRD guide cannot be designed for a single mode propagation like a rectangular waveguide. Moreover, the array is traveling wave fed, because $d \neq \lambda_g/2$.

Therefore, mutual coupling between Slots is severe in this case and its effect has to be included in the analysis. However, travelling wave antennas have typically good input impedance characteristics. The NRD guide supports fast waves for the propagating hybrid modes, which have guided wavelengths greater than the free space wavelength at the operating frequency. So, the Slot array in a NRD guide can be called a "fast wave" or "leaking wave" antenna. However,the phase velocity of the leaky mode differs little from its non-leaky counterpart.

A Slot array fed by a NRD guide is analysed using Scattering Matrix representation. Since, a NRD guide designed to excite PM_{11}

mode, also propagates PE_{11} and TE_{m0} modes, analyzing an array applying the Scattering Matrix approach is simpler. Microwave circuit theory mainly deals with input-output characteristics i.e. external characteristics. In most cases, the pattern and behaviour of internal electromagnetic fields are less important. For a single mode propagation, the back scattered and forward scattered wave amplitudes can be determined using the theory of Slot radiators.So, representing the Slot discontinuity by a multiport Scattering Matrix is sufficient to analyze the Slot array fed by a NRD guide. This can be viewed as an important advantage.

From the definition of the Scattering coefficients it follows that for multiport passive network, the magnitudes of the transmission and reflection coefficients $|S_{ij}|$, cannot exceed unity. The Scattering Matrix can describe the behaviour of the multiport network as a Microwave Circuit only at a specified frequency. If the terminal planes are located very close to the Slot discontinuity then the scattering coefficients are independent of frequency.

A Scattering Matrix gives complete the complete external characteristics of a multiport. It is well known, that different modes do not interact in a uniform waveguide, so for each mode the mutiport has a separate port and the Scattering Matrix has a dimension such that it takes into account all the modes propagating in the input leads. If the input leads of a multiport are sufficiently long, the non-propagating modes at the terminal planes may be neglected, and the multiport can be specified in terms of the Scattering Matrix that takes into account only the propagating modes. If, however, the elements are closely spaced, the interconnection between cascaded multiports is short, the higher-order non-propagating modes should

also be considered. The Scattering Matrix that takes into account all the propagating and non-propagating modes into account is called the Generalised Scattering Matrix.

In conclusion, an attractive feature of the Slot which is a commonly used radiator in Antenna Systems is that, it can be integrated into any array feed system without requiring a special matching network. To design a Slotted-Waveguide antenna array, the basic characteristics for a slot geometry, such as the radiated-field amplitude and phase including mutual-coupling effects between Slots and within the feed network, are needed. Equivalent impedance is the most convenient Slot characteristic, from the standpoint of Array design and simple analysis. However, the Scattering Matrix representation is required in the analysis and design of certain Slotted-Waveguide arrays in which complicated feed networks couple the Slots and there is multi-mode propagation in the guide .

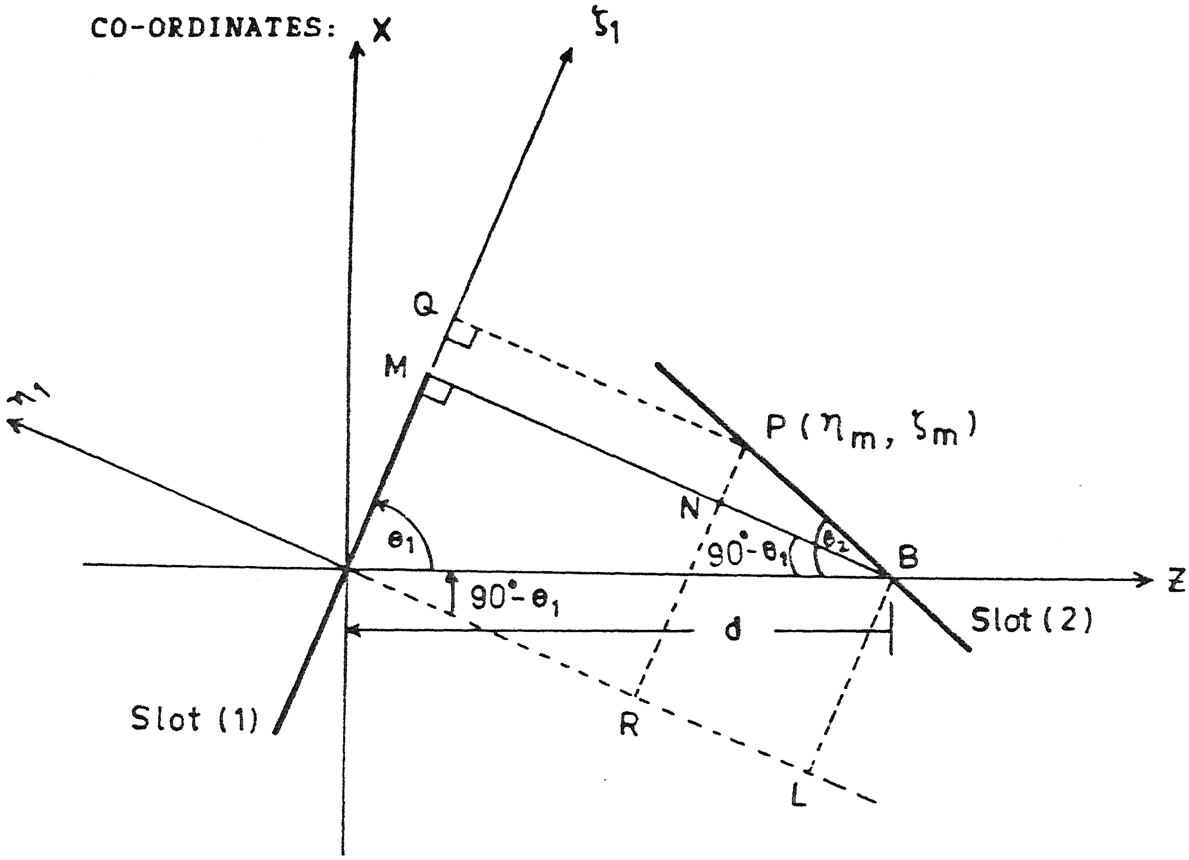
5.2 SCOPE FOR FURTHER WORK

The Generalised Scattering Matrix developed for a Slot and then extended to an N-element linear array can be used to design and test a Slot array in the NRD guide.

For PM_{11} mode excitation, the linear Slot array considered in this thesis gives rise to horizontal polarisation. If, along with the PM_{11} mode excitation another probe is used to excite PE_{11} mode, the polarisation obtained due to the same Slot array configuration is vertical. If the Slot orientation and other parameters involved adjusted to give equal magnitude for the electric field vector in both, the horizontal and vertical polarisations we can design the Slot array to get circular polarisation. This possible because of the orthogonality of PM_{11} and PE_{11} modes.

APPENDIX I

§ RELATION BETWEEN SLOT(1) CO-ORDINATES WITH THAT OF SLOT(2) CO-ORDINATES:



$AB = d$

$PB = \zeta_2$

$PQ = BM - BN$ and $BL = BL + PN$

$BN = PB\ COS(\theta_2+\theta_1 - 90^\circ) = PB\ COS(\theta_2+\theta_1)$

$PN = PB\ SIN(\theta_2+\theta_1 - 90^\circ) = -PB\ COS(\theta_2+\theta_1)$

Therefore,

$$\boxed{\begin{aligned}\eta_1 &= d \sin\theta_1 + \zeta_2 \sin(\theta_2-\theta_1)\\ \zeta_1 &= d \cos\theta_1 - \zeta_2 \cos(\theta_2-\theta_1)\end{aligned}}$$

because, θ_1 & θ_2 are opposite in direction.

By method of induction, the relation between the co-ordinates of the two Slots can be extended to a linear array consisting of N-Slots. Let us consider two arbitarry Slots n and m , which are adjacent to each other. The variables are related by

$$\boxed{\begin{aligned} \eta_n &= (m-n)d \sin\theta_n + \zeta_m \sin(\theta_m - \theta_n) \\ \zeta_n &= (m-n)d \cos\theta_n - \zeta_m \cos(\theta_m - \theta_n) \end{aligned}}$$

APPENDIX II

§ CALCULATION OF THE COUPLING COEFFICIENTS B_{ij} / C_{ij}

In sec(3.6), an example for the calculation of B_{52}, the coefficient, coupling PM_{11} mode with that of TE_{10} mode is explained. Eqn(3.9) is rewritten here for convenience

$$B_{ij} = \frac{\int_{slot} E_1 \times H_2 . dS}{2\int_{S_1} E_{qt} x H_{qt} . dS_1} \tag{A}$$

where, i=row of the matrix, denotes the coupled port

j=column of the matrix, denotes the incident port

To calculate B_{52}, the subscript 2 denotes that the incident mode is PM_{11} mode, and subscript 5 denotes that the coupled mode is TE_{10} mode. So, E_{qt} and H_{qt}, the incident mode fields at surface S_1 of the NRD guide correspond to the fields of PM_{11} mode and are given in table II. However, H_2 the magnetic field perpendicular to the Slot electric field distribution E_1 is the z-component of the TE_{10} and PM_{11} fields given in table III. E_1 corresponds to the PM_{11} mode.

Substituting all these equations in (A), we get the coupling coefficient B_{ij}. To calculate C_{ij} only the direction of propagation of the field component H_2 changes.

APPENDIX III

§ MATRICES FOR THE TRANSMISSION LINES INTERCONNECTING INDIVIDUAL SLOT MATRICES SHOWN IN FIG[4·3]

CONVERSION MATRIX

$$[\,S\,] = \begin{bmatrix} \dfrac{(A-D)+(B-C)}{D_A} & 2\Delta_A/D_A \\ 2/D_A & \dfrac{-(A-D)+(B-C)}{D_A} \end{bmatrix}$$

Where, Δ_A = AD-BC and D_A = A+B+C+D

The ABCD matrix for a transmission line of length "l", with β and Z_o as its characteristics:

$$[\,A\,] = \begin{bmatrix} \cos\beta l & jZ_o\sin\beta l \\ j/Z_o\ \sin\beta l & \cos\beta l \end{bmatrix}$$

In our case $l=\lambda/2$, frequecy of operation = 95Ghz

Therefore, l = 1.578947mm

For PM_{11} mode :

β = 884.3795 rad/mt $\qquad Z_o$ = 130.0894Ω

The ABCD matrix is:

$$[\,A\,] = \begin{bmatrix} .1735 & j128.1158 \\ j.00757 & .1735 \end{bmatrix}$$

For PE_{11} mode:

β = 1199.23 rad/mt $\qquad Z_o$ = 614.8087Ω

The ABCD matrix is:

$$|A| = \begin{bmatrix} -.3171 & j583.069 \\ j.00154 & -.3171 \end{bmatrix}$$

For TE_{10} mode:

β = 2637.284 rad/mt $\qquad Z_o$ = 614.8087Ω

$$|A| = \begin{bmatrix} -.5212 & -524.699j \\ -.00138j & -.5212 \end{bmatrix}$$

§ CALCULATION OF SCATTERING MATRIX ELEMENTS

i) V_{m1} is the maximum voltage at the center of the Slot resulting due to PM_{11} mode excitation:

f=95Ghz $\quad \theta=60^o \quad$ a=.0007839mm $\quad$ b=.0013375mm $\quad$ W=2mm

lower cutoff frequency = 90Ghz

Backscattered mode amplitude B11 = $.9892119\text{x}10^{-6}V_{m1}$,C=1

Let $P_{inc} = 1$

$A^{q1} = A11 = 2.11126\text{x}10^{-5}$

$$P_{refl} = 1/2\ T_{11}|B11|^2$$
$$= 2.1951\text{x}10^{-3}V_{m1}^2$$
$$P_{trans} = 1/2\ T_{11}|A^{q1}+C11|^2$$
$$= 1+P_{refl} + .093699V_{m1}$$
$$P_{rad} = 1.047047\text{x}10^{-3}V_{m1}^2$$

$$P_{inc} = P_{rad} + P_{refl} + P_{trans}$$

Therefore, $V_{m1} = -17.2328$ v

S_{11} = B11/A11 = -.8074

ii) V_{m2}, is the voltage at the center of the Slot due to PE_{11} mode excitation:

Following the same procedure and normalising the constant A to unity

$$B22 = .3461702\times10^{-3} V_{m2}$$

$$A^{q2} = A22 = 4.71118\times10^{-3}$$

$$P_{refl} = 5.39907 V_{m2}^{2} \times 10^{-3}$$

$$P_{trans} = 1 + P_{refl} + -.146956 V_{m2}$$

Therefore $\quad V_{m2} = 12.4063v$

$$S_{33} = B22/A22 = .9116$$

iii) V_{m3}, is the voltage at the center of the Slot due to TE_{10} mode excitation:

$$B33 = .642888\times10^{-6} V_{m3}$$ constant D is taken to be unity.

$$A^{q3} = A33 = 1.93083\times10^{-5}$$

$$P_{refl} = 1.1086\times10^{-3} V_{m3}^{2}$$

$$P_{trans} = 1 + P_{refl} + -.066355 V_{m3}$$

Therefore $\quad V_{m3} = 20.3276v$

$$S_{33} = B33/A33 = .6768$$

The overall Scattering Matrix for A single Slot fed by a NRD guide is:

$$\begin{bmatrix}
S_{11}\ -.8074 & S_{12} = S_{21} & S_{13}\ .9116 + j.6110 & S_{14} = S_{23} & S_{15}\ .6868 + j.8710 & S_{16} = S_{25} & S_{17} & S_{18}\ 0 & S_{19}\ 0 \\
S_{21}\ .1926 & S_{22} = S_{11} & S_{23}\ .0884 + j.6110 & S_{24} = S_{13} & S_{25}\ .3232 + j.8710 & S_{26} = S_{15} & S_{27} & S_{28}\ 0 & S_{29}\ 0 \\
S_{31}\ -.8074 - j5.674\times10^{-3} & S_{32} = S_{41} & S_{33}\ .9116 & S_{34} = S_{43} & S_{35}\ .6829 & S_{36} = S_{45} & S_{37}\ 0 & S_{38} & S_{39}\ 0 \\
S_{41}\ .1926 + j5.674\times10^{-3} & S_{42} = S_{31} & S_{43}\ .0884 & S_{44} = S_{33} & S_{45}\ .3171 & S_{46} = S_{35} & S_{47}\ 0 & S_{48} & S_{49}\ 0 \\
S_{51}\ -.8074 - j.6277 & S_{52} = S_{61} & S_{53}\ .4791 & S_{54} = S_{63} & S_{55}\ .6768 & S_{56} = S_{65} & S_{57}\ 0 & S_{58}\ 0 & S_{59} \\
S_{61}\ .1926 + j.6277 & S_{62} = S_{51} & S_{63}\ .5290 & S_{64} = S_{53} & S_{65}\ .3232 & S_{66} = S_{55} & S_{67}\ 0 & S_{68}\ 0 & S_{69} \\
S_{71} & S_{72} & S_{73}\ 0 & S_{74}\ 0 & S_{75}\ 0 & S_{76}\ 0 & S_{77}\ 0 & S_{78}\ 0 & S_{79}\ 0 \\
S_{81} = 0 & S_{82} = 0 & S_{83} & S_{84} & S_{85}\ 0 & S_{86} = 0 & S_{87}\ 0 & S_{88}\ 0 & S_{89}\ 0 \\
S_{91} = 0 & S_{92} = 0 & S_{93}\ 0 & S_{94}\ 0 & S_{95} & S_{96} & S_{97}\ 0 & S_{98}\ 0 & S_{99}\ 0
\end{bmatrix}$$

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